Calc AB

Lesson 98 | + 136

Agenda:
- Lesson 98
  - Fundamental Theorem of Calculus, Part 2 (98)
  - Using Chain Rule with FTC (136)

Recall: **FTC** f continuous on $[a,b]$. If F is any antiderivative of f then

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

★ FTC also guarentees that any continuous function has an antiderivative

$$\mathcal{F}(x) = \int_a^x f(t)\,dt \quad \text{is one antiderivative of } f(x)$$

$$\mathcal{F}(b) = \int_a^b f(t)\,dt = \text{area above - area below on } [a,b]$$

Check $\frac{d}{dx}\mathcal{F}(x) = \frac{d}{dx}\left(\int_a^x f(t)\,dt\right) \underset{\text{FTC}}{=} \frac{d}{dx}\left(F(x) - F(a)\right) \underset{\text{Def of } F}{=} f(x)$ ✓

FTC part 2

f continuous on $[a,b]$, $c \in [a,b]$ then f has an antiderivative

$$\mathcal{F}(x) = \int_c^x f(t)\,dt, \quad x \in [a,b]$$

Thus $\frac{d}{dx}\mathcal{F}(x) = \frac{d}{dx}\left(\int_c^x f(t)\,dt\right) = f(x)$

Ex. 98.1 Simplify $\frac{d}{dx}\int_a^x t^2\,dt = \boxed{x^2}$ by FTC part 2.

Ex. 98.2 Simplify ($x > 0$) $\frac{d}{dx}\int_x^4 \frac{\sin t}{t}\,dt = \frac{d}{dx} - \int_4^x \frac{\sin t}{t}\,dt = \boxed{-\frac{\sin x}{x}}$ $x > 0$ by FTC p2.

Skipppp!!!

Ex. Simplify $\frac{d}{dx}\int_x^{17} e^{-t^2}\,dt = -\frac{d}{dx}\int_{17}^x e^{-t^2}\,dt = \boxed{-e^{-t^2}}$

Ex. differentiate $\ln(x) = \int_1^x \frac{1}{t}\,dt$ $\quad \frac{1}{x} = \frac{d}{dx}\left(\int_1^x \frac{1}{t}\,dt\right) \Rightarrow \frac{1}{x} = \frac{1}{x}$

<u>Ex. 136.1</u> Simplify: $\frac{d}{dx}\left(\int_4^{2x} \sin(t^2)\,dt\right)$

$$h(u) = \int_4^{u} \sin(t^2)\,dt \quad \text{so} \quad h'(u) = \sin(u^2)$$

$$\frac{d}{dx} h(2x) = h'(2x)\cdot 2 = \boxed{2\sin(4x^2)}$$

<u>Ex. 136.2</u> Simplify: $\frac{d}{dx}\left(\int_{3x}^{\sin x} \cos(t^3)\,dt\right)$

$$h(u) = \int_0^{\sin x} \cos(t^3)\,dt \qquad g(u) = \int_0^{3x} \cos(t^3)\,dt$$

$$\frac{d}{dx}\left(\int_{3x}^{\sin x} \cos(t^3)\,dt\right) = \frac{d}{dx}\left(\int_0^{\sin x} \cos(t^3)\,dt - \int_0^{3x} \cos(t^3)\,dt\right)$$

$$= \frac{d}{dx} h(\sin x) - \frac{d}{dx} g(3x)$$

$$= \boxed{\cos(\sin^3 x)\cdot \cos x - \cos(27x^3)\cdot 3}$$