Calc AB

Lesson 97

Agenda:
Lesson 97
Solids defined by Cross Sections

★ Not all volumes are Solids of revolutions

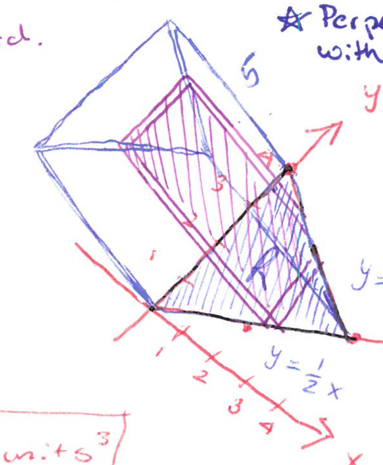
- look at solids with parallel cross sections all being the same simple geometric shape: square, circle, triangle

For disks: $\int_a^b \underbrace{\pi r^2}_{\text{area of disk top}} \overbrace{dx}^{\text{width}}$ represented a sum of volumes of disks

For Solids with parallel Cross sections all same shape

$$\text{Volume} = \int_a^b \underbrace{A(x)}_{\text{area of general shape}} \overbrace{dx}^{\text{width}}$$

Ex. 97.1 The base of a solid is the region R, bounded by $y = \frac{1}{2}x$, $y = -\frac{1}{2}x + 4$, and the y-axis. Every vertical cross section of the solid parallel to y-axis is a rectangle with height 5. Find the volume of the solid.



★ Perpendicular with vertical cross sections:
Semicircles
right isosceles triangles

Find area of general rectangle: base · height

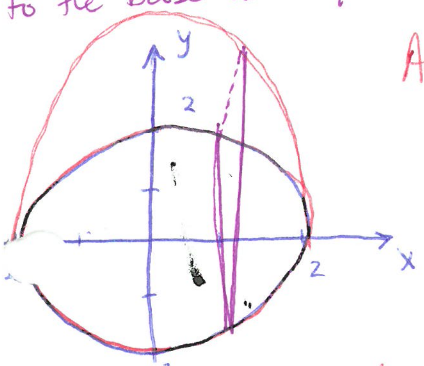
base $= \left(-\frac{1}{2}x + 4\right) - \left(\frac{1}{2}x\right) = -x + 4$

height $= 5$ $\quad A(x) = 5(4 - x)$

$$\text{Volume} = \int_0^4 A(x)\,dx = \int_0^4 20 - 5x\,dx$$

$$= 20(4) - \frac{5(4)^2}{2} = \boxed{40 \text{ units}^3}$$

Ex. 97.3 A solid has a circular base of radius 2. Vertical cross sections perpendicular to the base are equilateral triangles. Find the volume of the solid.

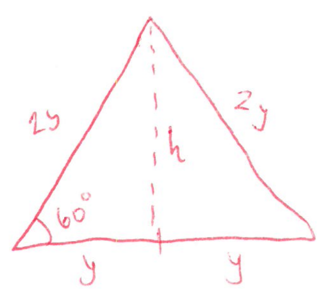


★ Cross sections parallel to y-axis means thickness is dx!

Area of triangle $= \frac{1}{2}(\text{base})(\text{height})$

$= \frac{1}{2}(2y)(\sqrt{3}y)$

$= \sqrt{3}y^2 = \sqrt{3}(4 - x^2)$

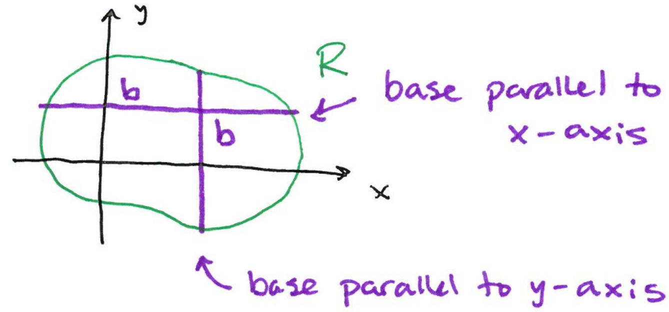


$h = 2y \sin 60° = \sqrt{3}y$

$$\text{Volume} = \int_{x=-2}^{x=2} \sqrt{3}(4 - x^2)\,dx = 2\sqrt{3}\int_0^2 4 - x^2\,dx$$

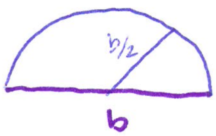
$$= 2\sqrt{3}\left(4(2) - \frac{2^3}{3}\right) = \boxed{\frac{32\sqrt{3}}{3} \text{ units}^3}$$

Volumes of Solids with Cross Sections:



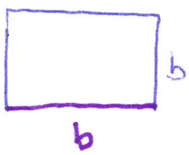
Base b in region R of xy-plane

- half circle with diameter b



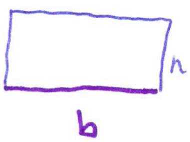
$A = \frac{\pi}{2}\left(\frac{b}{2}\right)^2 = \boxed{\frac{\pi}{8} b^2}$

- Square with side b



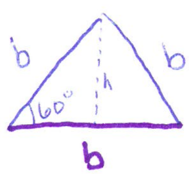
$\boxed{A = b^2}$

- Rectangle with base b, height h



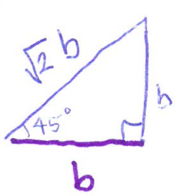
$\boxed{A = bh}$

- Equilateral Triangle with base b



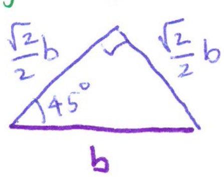
$A = \frac{1}{2} b \cdot h = \frac{1}{2} b \cdot b \sin 60° = \boxed{\frac{\sqrt{3}}{4} b^2}$

- Right isosceles triangle with leg base b



$A = \frac{1}{2} b \cdot b = \boxed{\frac{b^2}{2}}$

- Right isosceles triangle with hypotenuse base b



$A = \frac{1}{2}\left(\frac{\sqrt{2}}{2} b\right)\left(\frac{\sqrt{2}}{2} b\right) = \boxed{\frac{b^2}{4}}$