Calc AB

Agenda:

Lesson 96

Derivatives / Integrals of functions with absolute value

Recall: The derivative of $|f(x)|$ was $f'(x)$ when $f(x) > 0$ and $-f'(x)$ when $f(x) < 0$.

We write instead of a piecewise function:

$$\frac{d}{dx}|f(x)| = \underbrace{\frac{f(x)}{|f(x)|}}_{\text{indicated the sign}} \cdot f'(x)$$

Ex
$$\frac{d}{dx}|\sin x| = \frac{d}{dx}\begin{cases} \sin x & \text{if } \sin x > 0 \\ 0 & \text{if } \sin x = 0 \\ -\sin x & \text{if } \sin x < 0 \end{cases} = \begin{cases} \cos x & \text{if } \sin x > 0 \\ \text{undefined} & \text{if } \sin x = 0 \\ -\cos x & \text{if } \sin x < 0 \end{cases}$$

Or
$$\boxed{\frac{d}{dx}|\sin x| = \frac{\sin x}{|\sin x|} \cdot \cos x}$$

If we have instead $y = f(|x|) = \begin{cases} f(x) & \text{if } x \geq 0 \\ f(-x) & \text{if } x < 0 \end{cases}$

then
$$\frac{d}{dx} f(|x|) = \begin{cases} f'(x) & \text{if } x \geq 0 \\ -f'(-x) & \text{if } x < 0 \end{cases} = \boxed{\frac{x}{|x|} f'(|x|)}$$

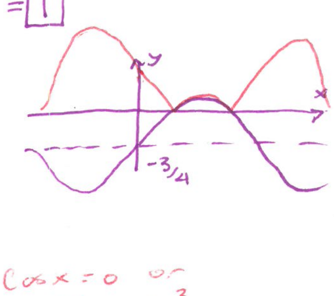
Ex. 96.1 Let $y = e^{|x|}$ find $\frac{dy}{dx}$.

$$\frac{dy}{dx} = \frac{x}{|x|} e^{|x|} = \begin{cases} e^{x} & \text{if } x > 0 \\ -e^{-x} & \text{if } x < 0 \end{cases}$$

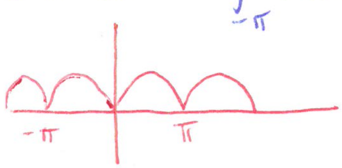
Ex. Find $\frac{d}{dx}(\sin|x|)$

$$= \frac{x}{|x|}\cos|x| = \begin{cases} \cos x & \text{if } x > 0 \\ -\cos(-x) & \text{if } x < 0 \end{cases}$$

Ex.
$$\int_{-\pi/2}^{0} \sin|x|\,dx = \int_{-\pi/2}^{0} \sin(-x)\,dx = \left(-\frac{\cos(-x)}{-1}\right)\Big|_{-\pi/2}^{0} = \cos(0) - \cos(\pi/2) = \boxed{1}$$



Ex. 96.5
$$\int_{-\pi}^{\pi}|\sin x|\,dx = 2\int_0^{\pi}\sin(x)\,dx = -2\cos(x)\Big|_0^{\pi} = 2 + 2 = \boxed{4}$$



Ex 96.6 Find the max value of $f(x) = |\sin(x) - \frac{3}{4}|$

Check cn's on $0 \leq x < 2\pi$ $\quad f'(x) = \frac{\sin x - \frac{3}{4}}{|\sin x - \frac{3}{4}|} \cdot \cos x \quad$ cn's: $\cos x = 0$ or $\sin x = \frac{3}{4}$

$x = \pi/2,\ 3\pi/2 \quad x = \sin^{-1}(\frac{3}{4}),\ \pi - \sin^{-1}(3/4) \quad f(\pi/2) = \frac{1}{4} \quad f(\frac{3\pi}{2}) = \frac{7}{4} \quad f(\sin^{-1}(\frac{3}{4})) = 0$

$f(\pi - \sin^{-1}(3/4)) = 0$