Calc AB

Lesson 94

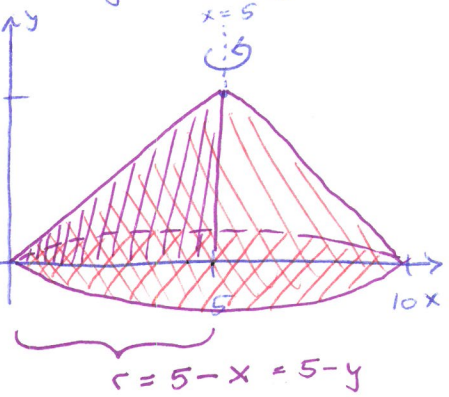
1/25/16

Agenda: 1/25/16
lesson 94
Solids of Revolution:
Axes of rev displaced

★ Handout Calendar

So far only done rotation about the x and y axis.

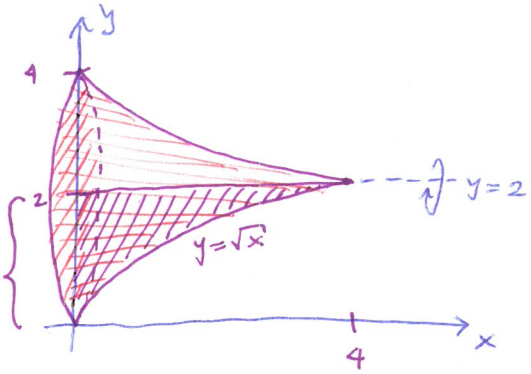
Ex. 94.1 Let R be the region bounded by the coordinate axes and the lines $y = x$ and $x = 5$. Find the volume of the solid formed by rotating R about $x = 5$.



$$\text{Volume} = \pi \int_{y=0}^{y=5} (5-y)^2 \, dy$$

$$= \pi \int_0^5 (25 - 10y + y^2)\, dy = \pi \left[25y - 5y^2 + \frac{y^3}{3}\right]_0^5 = \boxed{\frac{125\pi}{3}}$$

Ex. 94.3 R is completely enclosed by $x = y^2$, $y = 2$, and the y-axis. The region R is revolved about $y = 2$. Find the volume of the solid.

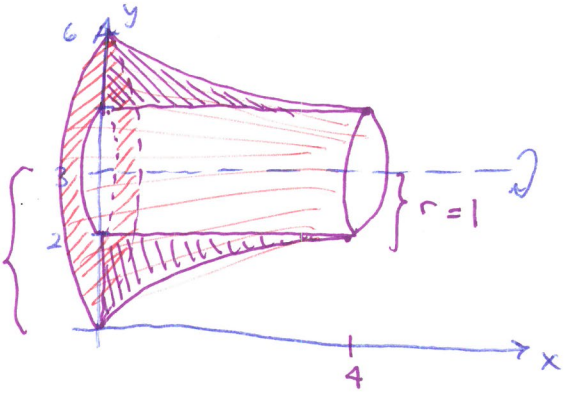


$$\text{Volume} = \pi \int_{x=0}^{x=4} (2-\sqrt{x})^2 \, dx$$

$$= \pi \int_0^4 (4 - 4\sqrt{x} + x)\, dx$$

$$= \pi \left(4x - \frac{8}{3}x^{3/2} + \frac{x^2}{2}\right)_0^4 = \pi \left[16 - \frac{64}{3} + 8\right] = \boxed{\frac{8\pi}{3}}$$

Ex. 94.4 Same region in 94.3 rotated about $y = 3$. Find the volume of the solid.



$$\text{Volume} = \pi \int_{x=0}^{x=4} (R^2 - r^2)\, dx$$

$$= \pi \int_0^4 (9 - 6\sqrt{x} + x - 1)\, dx$$

$$= \pi \left[8x - \frac{12}{3}x^{3/2} + \frac{x^2}{2}\right]_0^4$$

$$= \pi \left[32 - 32 + 8\right] = \boxed{8\pi \text{ units}^3}$$