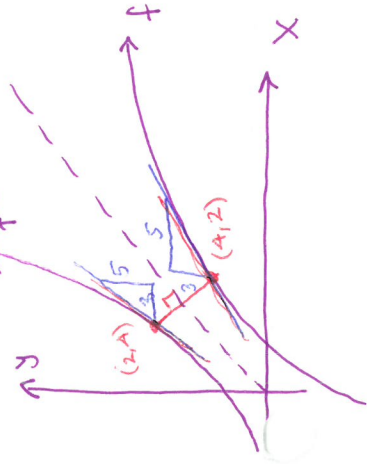


Agenda:

Lesson 9.2
Derivatives of Inverse FunctionsFind $\frac{d}{dx}(f^{-1}(x))$:Let $y = f^{-1}(x)$ then $f(y) = x$ [Property of Inverse]Differentiate both sides: $f'(y) \cdot \frac{dy}{dx} = 1$ so $\frac{dy}{dx} = \frac{1}{f'(y)}$ Replace $y = f^{-1}(x)$ so $\frac{d}{dx}(f^{-1}(x)) = \frac{1}{f'(f^{-1}(x))}$ 

$$f'(4) = \frac{3}{5}$$

$$(f^{-1})'(2) = \frac{5}{3} = \frac{1}{f'(4)} = \frac{1}{f'(f^{-1}(2))}$$

Ex 9.2.1 $f(x) = x^3 + x - 1$ is a 1-1 function, find the slope of the graph of the inverse f^{-1} at $(-1, 0)$.

$$\frac{d}{dx}(f^{-1}(x)) \Big|_{\substack{x=-1 \\ y=0}} = \frac{1}{f'(f^{-1}(x))} \Big|_{x=-1} = \frac{1}{f'(y)} \Big|_{y=0} = \frac{1}{3(0)^2 + 1} = 1$$

OR Find $(f^{-1})'$

$$x = y^3 + y - 1 \text{ so } 1 = 3y^2 \frac{dy}{dx} + \frac{dy}{dx} \text{ so } \frac{dy}{dx} = \frac{1}{3y^2 + 1} = \frac{d}{dx}(f^{-1}(x))$$

$$\text{So } \frac{d}{dx}(f^{-1}(x)) \Big|_{\substack{x=-1 \\ y=0}} = \frac{1}{3y^2 + 1} \Big|_{\substack{x=-1 \\ y=0}} = \frac{1}{3(0)^2 + 1} = 1$$

Ex: If $(2, 5)$ is a point on $f(x)$ and the slope of the graph of f at $x=2$ is $-\frac{1}{3}$ and if f has an inverse f^{-1} then what is the slope of the graph of f^{-1} at $x=5$?

$$\frac{d}{dx}(f^{-1}(x)) \Big|_{\substack{x=5 \\ y=2}} = \frac{1}{f'(f^{-1}(x))} \Big|_{\substack{x=5 \\ y=2}} = \frac{1}{f'(2)} = \frac{1}{-\frac{1}{3}} = -3$$