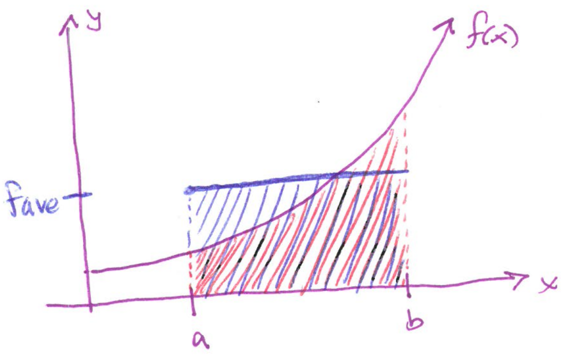
Calc AB

Lesson 89

Agenda:

- Lesson 89
  - Average Value of a function
  - Mean Value Theorem for Integrals



$f_{ave}(b-a) = \int_a^b f(x)\,dx$

So The average value of a continuous function on $[a,b]$ where $b>a$ is

$$\boxed{f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx}$$

Ex. 89.2 Find the average value of the function $f(x) = 4e^{2x}$ on $[0,3]$.

$$f_{ave} = \frac{1}{3}\int_0^3 4e^{2x}\,dx = \frac{4}{3}\left[\frac{e^{2x}}{2}\right]_0^3 = \frac{4}{3}\left(\frac{e^6}{2} - \frac{1}{2}\right)$$

$$= \frac{2}{3}(e^6 - 1) \approx \boxed{268.2859}$$

Mean Value Theorem for Integrals

If f is continuous on $[a,b]$ then there is at least one value c, $a<c<b$ such that

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx = f_{ave}$$

$$f(c) = \frac{F(b)-F(a)}{b-a}$$ F an antiderivative of f; $\frac{dF(x)}{dx} = f(x)$.

Ex. Use the Mean Value Theorem for Integrals to find some c such that $f(c) = f_{ave}$, if $f(x) = 6x^2 + 2x$ on $[0,2]$

$$6c^2 + 2c = f(c) = \frac{1}{2-0}\int_0^2 6x^2 + 2x\,dx$$

$$6c^2 + 2c = \frac{1}{2}\left(\frac{6x^3}{3} + \frac{2x^2}{2}\right)\Big|_0^2 = \frac{1}{2}(16+4) \Rightarrow 6c^2 + 2c = 10$$

$$c = \frac{-2 \pm \sqrt{4 - 4(-10)(6)}}{2(6)} = -\frac{1}{6} \pm \frac{\sqrt{61}}{6} \qquad \boxed{c \approx 1.135}$$

Ex. Find a value c such that $f(c) = f_{ave}$, if $f(x) = 2x^3$ on $[0,3]$.

$$2c^3 = \frac{1}{3-0}\int_0^3 2x^3\,dx = \frac{1}{3}\left(\frac{1}{2}x^4\right)\Big|_0^3 = \frac{27}{2} \Rightarrow c = \sqrt[3]{\frac{27}{4}} = \frac{3}{\sqrt[3]{4}}$$

$$\boxed{c \approx 1.8899}$$