Calc AB

Lesson 88

1/13/16

Agenda: 1/13/16
- Lesson 88
- Separable Differential Equations

A <u>Differential Equation</u> is an equation that contains one or more derivatives or differentials.

Examples: ① $\frac{dy}{dx} = x^2 + x \cdot \ln(x)$ ② $dy = 7x\cos(x)\,dx$

The Solution to a differential Equation is the set of all functions (or equations) which satisfy the differential equation.

To find the Solution:

① Guess/check Ex. $\frac{dy}{dx} = \sin(x)$ $\boxed{y = -\cos(x) + C}$

② Use different procedures mathematicians have developed based on the type of differential Equation.

First Type: Separable – if all terms with y can be moved to one side with all terms of x on the others. (i.e. separate x and y)

Ex. (1) $e^y \frac{dy}{dx} = x^2 \Rightarrow e^y\,dy = x^2\,dx$

Solve by integration: $\int e^y\,dy = \int x^2\,dx$

$\boxed{e^y = \frac{x^3}{3} + C}$ so $y = \ln\left(\frac{x^3}{3} + C\right)$ Called the General Solution to (1)

family of functions

☆ Only works for Separable Differential Equations!

If also given that $C = 5$ (for example)

then $y = \ln\left(\frac{x^3}{3} + 5\right)$ is Called a particular Solution to (1).

Ex. Find the general solution to $x\,dx - y^2\,dy = 0$.

$y^2\,dy = x\,dx \Rightarrow \frac{y^3}{3} = \frac{x^2}{2} + C \Rightarrow \boxed{y = \sqrt[3]{3\frac{x^2}{2} + 3C}}$

Ex. 88.5 Given the differential equation $\frac{dy}{dx} = 4x^2y^2$, find the particular solution y that passes through the point $(1,-1)$.

$$y^{-2}\,dy = 4x^2\,dx \Rightarrow \frac{y^{-1}}{-1} = \frac{4x^3}{3} + C$$

So $$y = \frac{-3}{4x^3 + 3C}$$

when $y=-1$, $x=1$:

$$-1 = \frac{-3}{4+3C} \Rightarrow -4-3C = -3$$

$$C = -\tfrac{1}{3}$$

So The particular Solution is $\boxed{y = \frac{-3}{4x^3 - 1}}$.

Ex. 88.2 The rate at which a certain bacteria colony is growing is proportional to the number of bacteria present at that time. At $t=0$ there are 1000 bacteria and at $t=1$ there are 1050 bacteria. Write an equation for the number of bacteria present at time t.

Pre-Calc Recall: $N(t) = P_0 e^{kt}$ but we'll use Calculus :)

$$\frac{dN}{dt} = kN \Rightarrow \frac{1}{N}\frac{dN}{dt} = k \Rightarrow \ln(N) = kt + C$$

So $$N(t) = e^{kt+C} = e^C \cdot e^{kt} = \boxed{Ae^{kt}}$$ Where A, C, k are constants.

To find A and k use:

$N(0) = 1000$ and $N(1) = 1050$

so $A = 1000$ $\qquad 1050 = 1000e^{k} \Rightarrow k = \ln\left(\frac{105}{100}\right) = \ln(1.05)$

$$\boxed{N(t) = 1000e^{\ln(1.05)\,t}}$$

Note: The rate at which something Y increases/decreases is proportional to the something Y at time t $\Big\}$ $\frac{dY}{dt} = kY$

The Solution is always $Y = Ae^{kt}$, A and k constants

[Good to know to save time]