

Agenda: 1/7/16

Lesson 86

Rules for Even and Odd Functions

★ HW 86 Due Monday

Recall: f is even if $f(-x) = f(x)$
 f is odd if $f(-x) = -f(x)$

- Sum of even functions is even
- Sum of odd functions is odd

Products: (even)(even) = (even) (odd)(odd) = (odd) (even)(odd) = odd

Quotients: $\frac{(\text{even})}{(\text{even})} = (\text{even})$ $\frac{(\text{odd})}{(\text{odd})} = (\text{odd})$ $\frac{(\text{odd})}{(\text{even})} = (\text{odd})$ $\frac{(\text{even})}{(\text{odd})} = (\text{even})$

Ex. Let f, g be odd functions. Show (fg) is an even function.

$$(fg)(-x) = f(-x)g(-x) = (-f(x)) \cdot (-g(x)) = fg(x) \Rightarrow \text{Even} \checkmark$$

Ex. Show that $\frac{f}{g}$ is odd if f is even and g is odd.

$$\frac{f}{g}(-x) = \frac{f(-x)}{g(-x)} = \frac{f(x)}{-g(x)} = -\frac{f}{g}(x) \Rightarrow \text{odd} \checkmark$$

Ex. Let f be even with $\int_3^{12} f(x) dx = -8$. What is $\int_{-12}^{-3} f(x) dx$?

$$\int_{-12}^{-3} f(x) dx = \int_3^{12} f(-x) dx = \int_3^{12} f(x) dx = \boxed{-8}$$

Ex. 86.5 If $\int_{-1}^1 e^{x^2} dx = K$, then what is the value of $\int_0^1 e^{x^2} dx$?

$$f(x) = e^{x^2} \text{ is even so } \int_0^1 e^{x^2} dx = \frac{1}{2} \int_{-1}^1 e^{x^2} dx = \boxed{\frac{K}{2}}$$