Calc 4B | Lesson 85 | 1/11/16

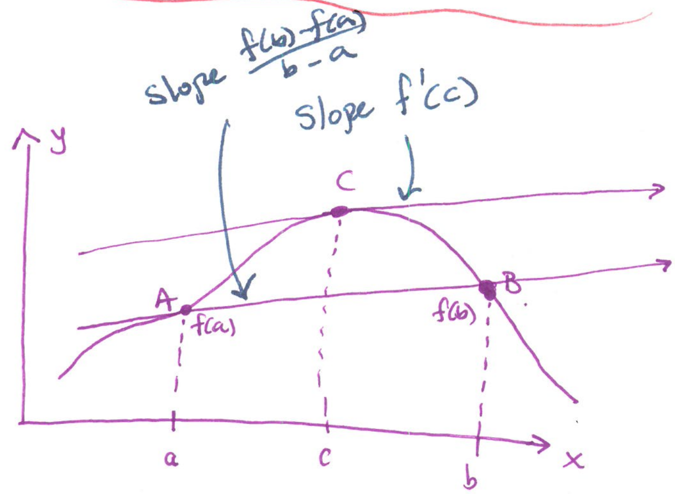
Agenda: 1/11/16
- lesson 85
  - Mean Value The[orem] (MVT)
  - Rolle's Theorem

Recall (Maybe)

EVT: Continuous f on $[a,b]$ attains an absolute max and min.

IVT: Continuous f on $[a,b]$, N between $f(a)$, $f(b)$ $\exists$ $a<c<b$ s.t. $f(c)=N$.

Existence Theorem:



Mean Value Theorem:

f continuous on $[a,b]$ and f is differentiable on (a,b), then there exists at least one c in (a,b) such that

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

★ Doesn't Find C for You!!

Ex. Apply MVT to $f(x) = x^3 - 5$ on $[-1, 2]$

(1) Continuous? $\boxed{\text{Yes}}$ (2) Differentiable on $(-1,2)$? $\boxed{\text{Yes}}$ so MVT Applies

So There exists a c in $(-1,2)$ with

$$f'(c) = \frac{f(2)-f(-1)}{2-(-1)} = \frac{3-(-6)}{3} = 3 \quad \Big] \text{ MVT}$$

Now $f'(x) = 3x^2 - 5$ So $3 = 3x^2 - 5 \Rightarrow x = \pm\sqrt{\frac{8}{3}}$

Ex. The value of $g(x) = \frac{1}{x^2} - \frac{1}{4}$ equals zero at $x = \pm 2$. The slope of the line connecting $(-2,0)$ and $(2,0)$ is zero. Can MVT be used to prove the existence of a point in $(-2,2)$ for which the slope of the tangent line is zero?

$\boxed{\text{No}}$ $g(x)$ is not continuous on $[-2,2]$ because there is an asymptote at $x=0$.

Ex. 85.4 $f(x) = |x| - 1$ is equal to 0 at $x=-1$ and $x=1$. Does MVT imply that $f'(c) = 0$ for some c in $(-1,1)$?

(1) $f(x)$ is continous on $[-1,1]$ ✓

(2) $f(x)$ is not differentiable on $(-1,1)$ since $f'(0)$ DNE.

No Can't apply MVT.

Applications of MVT

★ If $f'(x) = 0$ on $[a,b]$ then $f(x) = C$ on $[a,b]$ for some constant C.

Proof: let $a \le x_1 < x_2 \le b$ then by MVT $\exists$ $x_1 < c < x_2$ with

$$0 = f'(c) = \frac{f(x_2) - f(x_1)}{x_2 - x_1} \Rightarrow f(x_2) = f(x_1)$$

As this holds for all $a \le x_1 < x_2 \le b \Rightarrow f(x) = C$ for some C. ∎

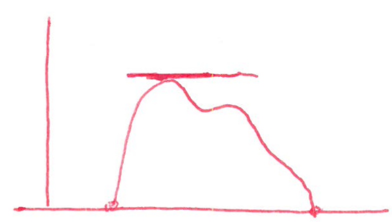
★ If $f'(x) = g'(x)$ on $[a,b]$ then $f(x) - g(x) = C$ on $[a,b]$.

Proof: $f'(x) - g'(x) = 0$ on $[a,b]$

$(f-g)'(x) = 0$ so by first application $f(x) - g(x) = C$ on $[a,b]$ ∎

Rolle's Theorem (Special Case of MVT)

f continuous on $[a,b]$, $f(a) = f(b) = 0$, and f differentiable on (a,b) then $\exists$ $a < c < b$ with $f'(c) = 0$.



Practical Application: SPEED CAMERAS

- Appear in the UK mainly
- Common Camera (in USA) measure instaneous speed as you pass
- SPECS Cameras on freeways in UK measures average speed $= \frac{\text{distance traveled}}{\text{time traveled}}$

Ex. At 5:00 PM a police officer sees a car go 70 milesperhour on a 75 mph highway. At 6:00 PM, 90 miles down the road a second officer sees the same car going 60 mph. The driver is ticketed for speeding but he argues that he was never clocked above 70 mph. Was he right to be ticketed?

Average speed $= \frac{90 \text{ miles}}{1 \text{ hr}} = 90$ mph by MVT $\exists$ a time between 5 and 6 PM when he was going 90 mph!