Calc AB

Lesson 82

1/5/16

Agenda: 1/5/16

Differentiability

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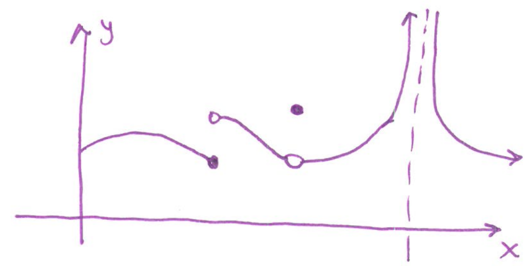
Recall: $f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$ provided the limit exists

★ For a function to have a derivative at $x = c$ it is necessary that the function be continuous at $x = c$.




i.e. If f is differentiable at $x = c$,

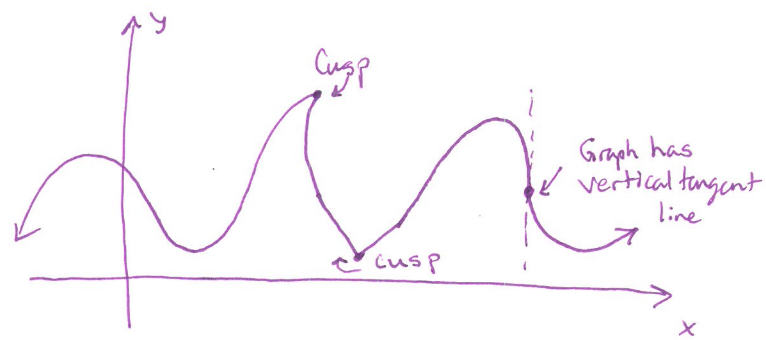
then f is continuous at $x = c$.

$\boxed{f'(c) \text{ exists then } \lim_{x \to c} f(x) = f(c)}$

★ If a function is continuous but a derivative does not exist at a point then the graph of the function comes to a sharp point or has a vertical tangent line.

The values of x where the derivative doesn't exist are called singular numbers.

Example 82.2




For what values of x is the piecewise function shown below differentiable?

$$f(x) = \begin{cases} x^2 & \text{if } x \geq 1 \\ 2x - 1 & \text{if } x < 1 \end{cases}$$

Each piece is differentiable so just need to check over lap.

Need $\lim_{x \to 1^+} f'(x) = \lim_{x \to 1^-} f'(x)$

$\lim_{x \to 1^+} f'(x) = \lim_{x \to 1^+} 2x = 2$

$\lim_{x \to 1^-} f'(x) = \lim_{x \to 1^-} 2 = 2$

Thus f is differentiable for all Values of x.

Example 82.3 Let f be defined as

$$f(x) = \begin{cases} |x| + 3 & \text{if } x < 1 \\ ax^2 + bx & \text{if } x \geq 1 \end{cases}$$

Find the values of a and b s.t. f is continuous and differentiable at $x = 1$.

① Continuous: $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^-} f(x) = f(1) = a + b$

So $\boxed{a + b = 4}$

② Differentiable: $\lim_{x \to 1^+} f'(x) = \lim_{x \to 1^-} f'(x)$

So $\lim_{x \to 1^+} \frac{d}{dx}(|x| + 3) = \lim_{x \to 1^-} \frac{d}{dx}(ax^2 + bx)$

$\boxed{1 = 2a + b}$

$\boxed{a = -3 \quad \text{and} \quad b = 7}$