Calc AB

Lesson 81

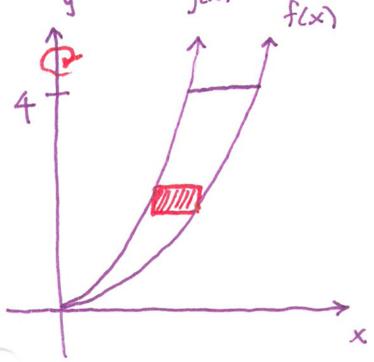
1/4/16

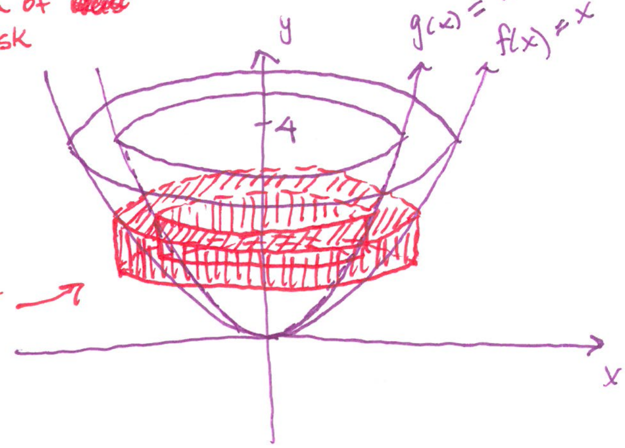
Agenda : 1/4/16

- Lesson 81
  - Solids of Revolutions II : Washers
- ★ Handout Calendar
- ★ Handout Solids of rev WS 2

Solids of Revolutions with two curves – hallow bowls or holes

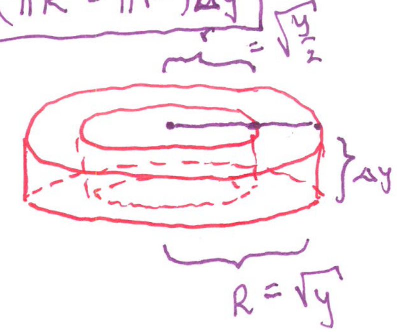
Recall: Volume $= \int_a^b \pi r^2 \, dx$ — πr^2 = area of disk; $\pi r^2\,dx$ = Volume of disk





Volume of one washer:

$$(\pi R^2 - \pi r^2)\Delta y$$



$$\text{Volume} = \int_a^b (\pi R^2 - \pi r^2)\,dy \quad \text{revolved about the y-axis}$$

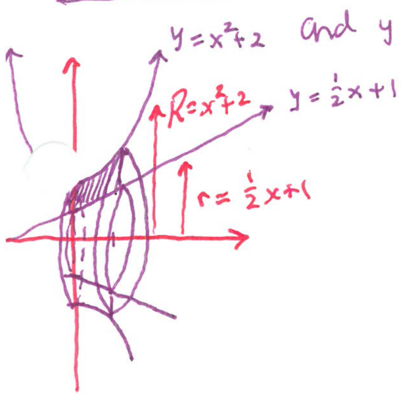
$$\text{Volume} = \int_a^b (\pi R^2 - \pi r^2)\,dx \quad \text{revolved about the x-axis}$$

Example 81.1 Find the volume of the solid formed by revolving the above region about the y-axis.

$R^2 = x^2 = y \qquad r^2 = x^2 = \frac{y}{2} \qquad \text{Volume} = \int_0^4 \left(\pi y - \pi \frac{y}{2}\right) dy$

$$= \pi \int_0^4 \frac{y}{2}\,dy = \pi \left.\frac{y^2}{4}\right|_0^4 = \boxed{4\pi \text{ units}^3}$$

Example 81.4 Find the volume of the solid formed by revolving the region between $y = x^2 + 2$ and $y = \frac{1}{2}x + 1$ on $[0,1]$ about the x-axis.



$$\text{Volume} = \int_0^1 \left(\pi (x^2+2)^2 - \pi\left(\tfrac{1}{2}x+1\right)^2\right) dx$$

$$= \int_0^1 \pi\left[(x^4+4x^2+4) - \left(\tfrac{1}{4}x^2 + x + 1\right)\right] dx$$

$$= \pi\left[\frac{x^5}{5} + \frac{4}{3}x^3 + 4x - \frac{1}{12}x^3 - \frac{x^2}{2} - x\right]\Bigg|_0^1$$

$$= \pi\left[\frac{1}{5} + \frac{4}{3} + 4 - \frac{1}{12} - \frac{1}{2} - 1\right]$$

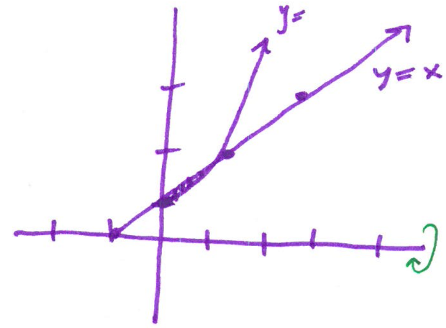
$$= \pi\left[\frac{1}{5} + \frac{3}{4} + 3\right]$$

$$= \boxed{\frac{79\pi}{20} \text{ units}^3}$$

Lesson 81 - Volumes of Solids of Revolutions - Washers

Ex. Find the volume of the solid of revolution of the region

$y = x + 1$, $y = x^3 + 1$, $x \geq 0$



revolved about

a) x-axis

$x^3 + 1 = x + 1$

$x = \pm 1, 0$

$$\pi \int_0^1 (x+1)^2 - (x^3+1)^2 \, dx$$

$$= \pi \int_0^1 x^2 + 2x + 1 - x^6 - 2x^3 - 1 \, dx$$

$$= \pi \int_0^1 -x^6 - 2x^3 + x^2 + 2x \, dx$$

$$= \boxed{\pi \left[-\frac{1}{7} - \frac{1}{2} + \frac{1}{3} + 1 \right]} = \boxed{\frac{29\pi}{42}}$$

b) y-axis

$$\pi \int_1^2 (y-1)^{2/3} - (y-1)^2 \, dy =$$

$$= \pi \left[(y-1)^{5/3} \cdot \frac{3}{5} - \frac{(y-1)^3}{3} \right] \Bigg|_1^2$$

$$= \boxed{\pi \left[\frac{3}{5} - \frac{1}{3} \right]} = \boxed{\frac{4\pi}{9}}$$