Calc AB

Lesson 78

12/10/15

Agenda: 12/10/15
lesson 78
Particle Motion I
* Quiz 8 tomorrow

- Free-falling body problems acceleration is constant $-9.8\ m/s^2$ (or $-32\ ft/s^2$)
- look at particles that move left and right with an acceleration that is a function of time.

Horizontal motion but graph vertically

Ex. 78.2 A particle moves along the x-axis such that the acceleration function is $a(t) = -3t$. Its position is 20 units when $t=3$ and its velocity is 5 when $t=1$. What is the position when $t=4$ and at what times is the particle changing direction?

$$v(t) = \int a(t)\,dt = -\frac{3t^2}{2} + C \qquad v(1)=5 \text{ so } 5 = -\frac{3}{2} + C \Rightarrow C = \frac{13}{2}$$

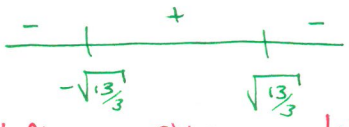
$$x(t) = \int v(t)\,dt = \int \left(-\frac{3t^2}{2} + \frac{13}{2}\right)dt = -\frac{t^3}{2} + \frac{13}{2}t + C \qquad x(3)=20 \quad 20 = -\frac{27}{2} + \frac{39}{2} + C$$

$$x(t) = -\frac{t^3}{2} + \frac{13}{2}t + 14 \qquad x(4) = -\frac{64}{2} + 26 + 14 = \boxed{8} \qquad C = 14$$

Changing direction when $v(t)$ changes sign:

$v(t)=0$ when $\boxed{t = \pm\sqrt{\frac{13}{3}}}$

Sign of $v(t)$:



Ex 78.3

A particle moves along the x-axis so that its velocity at time t is given by $v(t) = \frac{1}{t}$ for $t>0$, and its position is 5 when $t=2$. Find the time when the particle is 10 units to the right of the origin.

$$x(t) = \int \frac{1}{t}\,dt = \ln|t| + C \qquad x(t) = \ln(t) + C \text{ for } t>0$$

$$5 = x(2) = \ln(2) + C \qquad \boxed{x(t) = \ln(t) + 5 - \ln(2)}$$

$$10 = \ln(t) + 5 - \ln(2) \qquad \ln(t) = 5 + \ln(2) \qquad \boxed{t = e^{5+\ln(2)} = 2\cdot e^5 \approx 296.82}$$