Calc AB

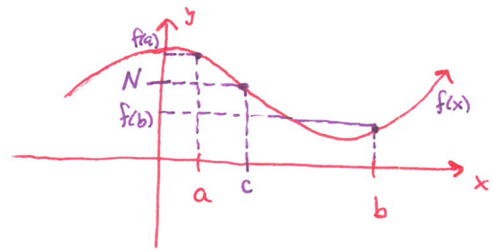
lesson 75

12/9/15

Agenda: 12/9/15
- lesson 75
  - Continuity of functions
- ★ Quiz 8 on Friday 66-73

Obvious but useful:

Intermediate Value Theorem: f continuous on $[a,b]$ and N is a number between $f(a)$ and $f(b)$, then there is at least one number c between a and b, inclusive, for which $f(c) = N$.



Ex. 75.1 Prove that $f(x) = x^3 - 5x + 2$ has a root between $x = 0$ and $x = 1$.

f is continuous over $[0,1]$ and $f(0) = 2$ and $f(1) = -2$. Since 0 is between $f(0) = 2$ and $f(1) = -2$ then by the intermediate value theorem there is at least one value between 0 and 1, c with $f(c) = 0$. Thus f has a root between $x = 0$ and $x = 1$.

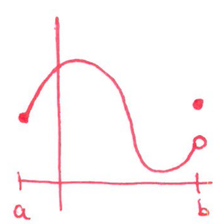
★ IVT doesn't locate the value c only gives its existence.

Continuity is Important lets recall the definition:

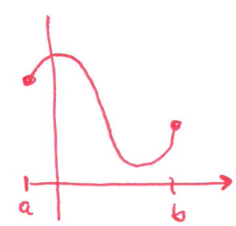
Continuity of a function at a point: A function f is continuous at $x = c$ if
1. $f(c)$ exists
2. $\lim_{x \to c} f(x) = f(c)$ $\quad \left[\lim_{x \to c^+} f(x) = \lim_{x \to c^-} f(x) = f(c) \right]$

Open Interval Continuity: A function f is continuous on an open interval (a,b) if it is continuous at every point on the interval.

Closed Interval Continuity: A function f is continuous on a closed interval $[a,b]$ if it is continuous on (a,b) and if both
1. $f(a)$ and $f(b)$ exist
2. $\lim_{x \to a^+} f(x) = f(a)$ and $\lim_{x \to b^-} f(x) = f(b)$.



Not Continuous on $[a,b]$



Continuous on $[a,b]$

Ex. 75.4 Let $f(x) = \begin{cases} |x| + 3 & \text{if } x < 1 \\ ax^2 + bx & \text{if } x \geq 1 \end{cases}$ Find a, b such that f is continuous on $(-\infty, \infty)$.

Each piece of f is continuous so the only problem would happen at the transition. $f(1) = a + b$ so it exists. Also need $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^-} f(x) = f(1)$ so $\boxed{a + b = |1| + 3 = 4}$

Ex. 75.5 Let, Is f continuous on $(-\infty, \infty)$?

$f(x) = \begin{cases} \frac{x^2 - c^2}{x + c} & \text{when } x \neq -c \\ -2c & \text{when } x = -c \end{cases}$

Only potential problem at $x = -c$. $f(-c) = -2c$ exists

$\lim_{x \to -c} f(x) = \lim_{x \to -c} \frac{(x-c)(x+c)}{x+c} = \lim_{x \to -c} (x - c) = -2c = f(-c)$

$\boxed{\text{Thus } f(x) \text{ is continuous on } (-\infty, \infty).}$