Calc AB

Lesson 73

12/8/15

Agenda: 12/8/15
Lesson 73
Integrals of a^x and $\log_a(x)$

Recall: $\frac{d}{dx}(a^x) = a^x \cdot \ln(a)$

Thus $\boxed{\int a^x \, dx = \frac{a^x}{\ln(a)} + C}$ because $\frac{d}{dx}\left(\frac{a^x}{\ln(a)}\right) = a^x$

$\boxed{\int \ln(x)\, dx = x \cdot \ln(x) - x + C}$ [Integration by parts seen in BC]

because $\frac{d}{dx}(x\ln(x) - x) = \ln(x) + \frac{x}{x} - x = \ln(x)$

Thus $\int \log_a(x)\, dx = \int \frac{\ln(x)}{\ln(a)}\, dx = \boxed{\frac{1}{\ln(a)}(x\ln(x) - x) + C}$

Ex. Integrate: $\int 5x\, a^{3x^2-7}\, dx$

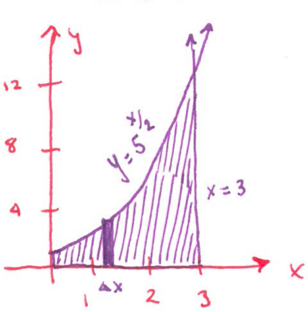
$u = 3x^2 - 7$
$du = 6x\, dx$

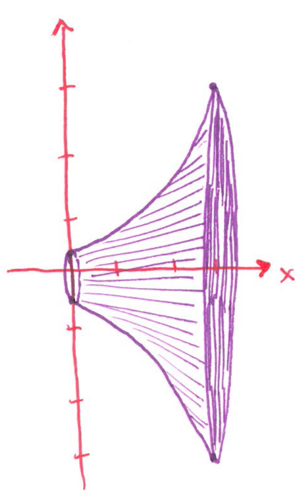
$= \int 5x\, a^u \cdot \frac{du}{6x}$

$= \frac{5}{6}\int a^u\, du$

$= \boxed{\frac{5}{6} \cdot \frac{1}{\ln(a)} \cdot a^{3x^2-7} + C}$

Ex. 73.3 Find the volume of the solid obtained by rotating about the x-axis the region bounded by the graphs of $y = 5^{x/2}$, $x = 3$ and the coordinate axes.





$\text{Volume} = \int_0^3 \pi \left(5^{x/2}\right)^2 dx = \pi\int_0^3 5^x\, dx$

$= \pi\left[5^x \cdot \frac{1}{\ln(5)}\right]\Big|_0^3$

$= \boxed{\frac{\pi}{\ln(5)} \cdot 124 \text{ units}^2}$