Calc AB

# Lesson 72

12/7/15

Agenda: 12/7/15
- Lesson 72
  - Derivatives of $a^x$, $\log_a x$, $|f(x)|$

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Compute $\frac{d}{dx}(\log_a x)$: Only know $\frac{d}{dx}(\ln(x)) = \frac{1}{x}$ so we'll use change of base formula

$\log_a x = \frac{\ln(x)}{\ln(a)}$ so $\frac{d}{dx}(\log_a x) = \frac{d}{dx}\left(\frac{\ln(x)}{\ln(a)}\right) = \frac{1}{\ln(a)\cdot x}$

$$\boxed{\frac{d}{dx}(\log_a x) = \frac{1}{x\cdot\ln(a)}}$$

Compute $\frac{d}{dx}(a^x)$: Use $\log_a(a^x) = x$ and implicit differentiation

$\frac{d}{dx}(\log_a(a^x)) = 1$ so $\frac{1}{\ln(a)\cdot a^x}\cdot\frac{d}{dx}(a^x) = 1$

$$\boxed{\frac{d}{dx}(a^x) = a^x\ln(a)}$$

[OR use $\frac{d}{dx}(e^x) = e^x$
$a^x = e^{\ln(a^x)} = e^{x\ln(a)}$
$\frac{d}{dx}(a^x) = e^{x\cdot\ln(a)}\cdot\ln(a) = a^x\cdot\ln(a)$

Ex. 72.5 let $f(x) = \log_9(x^2 + \sin(x))$. Approximate the slope of the tangent line to f at $x=1$.

$f'(x) = \frac{2x + \cos(x)}{\ln(9)(x^2 + \sin(x))}$ $\quad f'(1) = \frac{2 + \cos(1)}{\ln(9)(1 + \sin(1))} \approx \boxed{0.6278}$

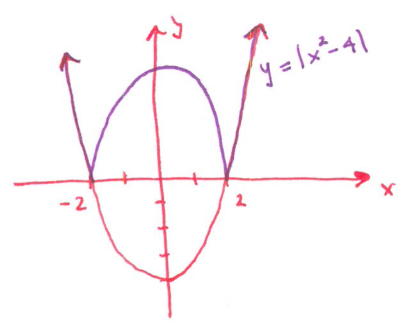
Compute $\frac{d}{dx}(|f(x)|)$: Recall $\frac{d}{dx}(|x|) = \begin{cases} -1 & \text{if } x<0 \\ DNE & \text{if } x=0 \\ 1 & \text{if } x>0 \end{cases}$

$\frac{d}{dx}|f(x)| = \begin{cases} -f'(x) & \text{if } f(x)<0 \\ DNE & \text{if } f(x)=0 \\ f'(x) & \text{if } f(x)>0 \end{cases}$

$\frac{d}{dx}(|x-5|) = \begin{cases} -1 & \text{if } x<5 \\ DNE & \text{if } x=5 \\ 1 & \text{if } x>5 \end{cases}$ because $y = |x-5| = \begin{cases} 5-x & \text{if } x<5 \\ 0 & \text{if } x=5 \\ x-5 & \text{if } x>5 \end{cases}$

Ex. 72.8 let $y = |x^2 - 4|$ find $y'$.

$y = \begin{cases} x^2-4 & \text{if } |x|>2 \\ 0 & \text{if } |x|=2 \\ 4-x^2 & \text{if } |x|<2 \end{cases}$ $\qquad y' = \begin{cases} 2x & \text{if } |x|>2 \\ DNE & \text{if } |x|=2 \\ -2x & \text{if } |x|<2 \end{cases}$




$\lim_{|x|\to 2^+} y' = \lim_{|x|\to 2^+} 2x = \underline{4}$ $\quad \lim_{|x|\to 2^-} -2x = \underline{-4}$

$\Rightarrow \lim_{|x|\to 2} y' = DNE$