Calc AB

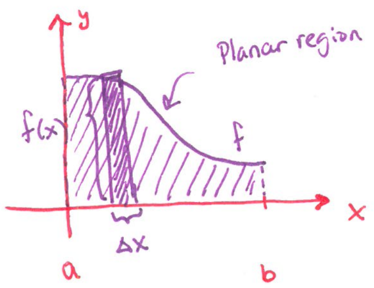
Lesson 71

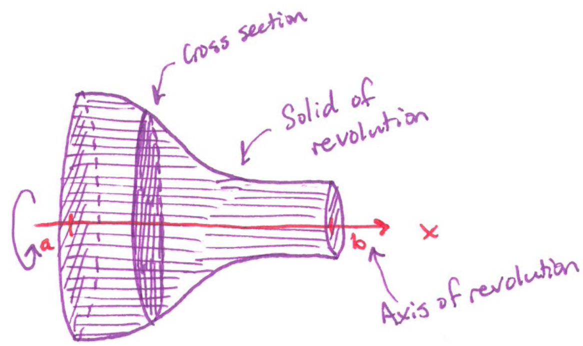
12/3/15
+ 12/4/15

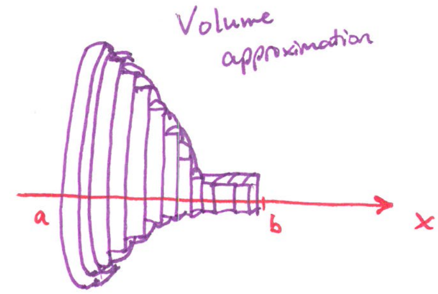
Agenda: 12/3/15
Lesson 71
Solids of revolution I: Disks

★ Handout WS on Solids I

A planar region revolved about a line in the same plane forms a figure called a solid of revolution. The line is the axis of revolution.







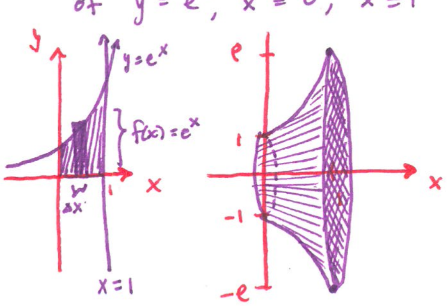
★ Right now we'll look at solids with circular cross sections, rotated about x or y axis.

Volume can be approximated by the sum of n circular disks of area πr^2 and thickness Δx.

$$\text{Volume Approx.} = \sum_{i=1}^{n} \pi \left[f(x_i) \right]^2 \Delta x \qquad \text{Exact Volume} = \int_a^b \pi \left[f(x) \right]^2 dx$$

Example 71.2

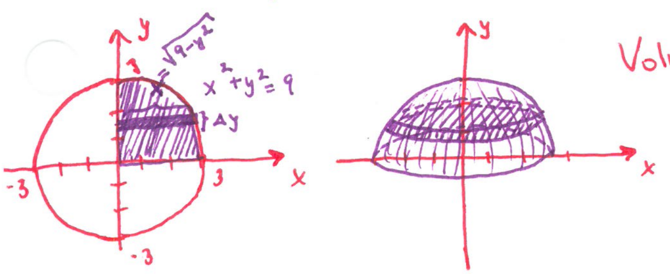
Find the volume of the solid formed by revolving about the x-axis the region bounded by the graphs of $y = e^x$, $x = 0$, $x = 1$ and the x-axis.



$$\text{Volume} = \int_0^1 \pi \left[e^x \right]^2 dx = \pi \int_0^1 e^{2x}\, dx = \pi \left[\frac{1}{2} e^{2x} \right]\Big|_0^1 = \boxed{\frac{\pi}{2}\left[e^2 - 1 \right] \text{units}^2}$$

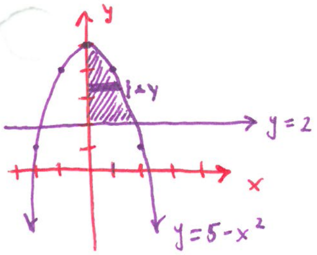
Example 71.4

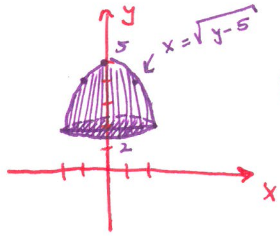
Find the volume of the solid formed by rotating about the y-axis the first quadrant region of the circle whose equation is $x^2 + y^2 = 9$.



$$\text{Volume} = \int_0^3 \pi \left[\sqrt{9-y^2} \right]^2 dy = \pi \int_0^3 (9-y^2)\, dy = \pi \left[9y - \frac{y^3}{3} \right]_0^3 = \boxed{18\pi \text{ units}^2}$$

Example: Find the volume of the solid formed by rotating about the y-axis the region bounded by the graphs of $y = 5 - x^2$, $y = 2$ and the y-axis.



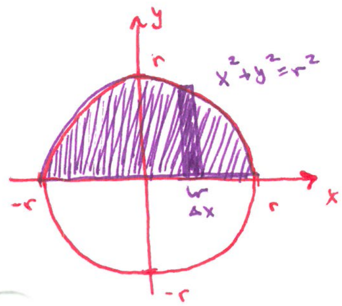


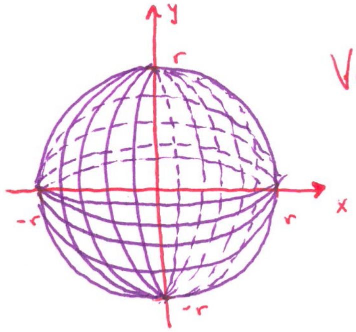
$$\text{Volume} = \int_2^5 \pi\left[\sqrt{5-y}\right]^2 dy = \pi\int_2^5 (5-y)\,dy = \pi\left[5y - \frac{y^2}{2}\right]_2^5$$

$$= \pi\left[25 - \frac{25}{2}\right] - \pi\left[10 - 2\right]$$

$$= \boxed{\frac{9}{2}\pi \text{ units}^2}$$

Example: Find the Volume of a sphere of radius r by rotating the region bounded above by $x^2 + y^2 = r^2$ and below by the x-axis about the x-axis and compute its volume.





$$\text{Volume} = \int_{-r}^{r} \pi\left[\sqrt{r^2 - x^2}\right]^2 dx$$

$$= 2\pi\int_0^r (r^2 - x^2)\,dx$$

$$= 2\pi\left(r^2x - \frac{x^3}{3}\right)_0^r$$

$$= 2\pi\left(\frac{2}{3}r^3\right) = \boxed{\frac{4}{3}\pi r^3 \text{ units}^2}$$