Calc AB

# Lesson 67

11/12/15

Agenda: 11/12/15

Lesson 67

Areas involving Functions of y

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★ y has been a function of x but for some problems it is very convenient to have x a function of y.

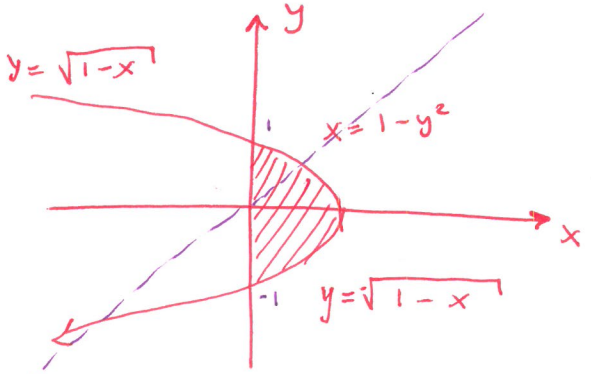
**Ex 67.1** Find the area of the region completely enclosed by the y-axis and the graph of $x = 1 - y^2$.

$$\text{Area of region} = \int_{-1}^{1} (1 - y^2)\,dy$$

$$= y - \frac{y^3}{3}\Big|_{-1}^{1}$$

$$= \left(1 - \frac{1}{3}\right) - \left(-1 + \frac{1}{3}\right)$$

$$= \boxed{\frac{4}{3}} \text{ units}^2$$




$$\int_0^1 2\sqrt{1-x}\,dx = -2(1-x)^{3/2} \cdot \frac{2}{3}\Big|_0^1$$

$$= \frac{4}{3}(1) = \boxed{\frac{4}{3}}$$

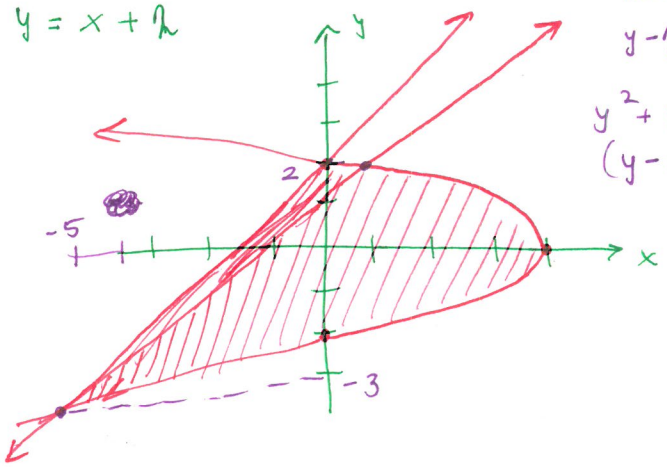
**Ex** Find the area of the region completely bounded by the graphs of $x = 4 - y^2$ and $y = x + 1$.

Bounds:

$y - 1 = 4 - y^2$

$y^2 + y - 6 = 0$

$(y - 2)(y + 3) = 0$




$$\text{Area of region} = \int_{-3}^{2} (\text{Right} - \text{Left})\,dy$$

$$= \int_{-3}^{2} (4 - y^2 - (y - 2))\,dy$$

$$= \left[4y - \frac{y^3}{3} - \frac{y^2}{2} + 2y\right]\Big|_{-3}^{2}$$

$$= \left(12 - \frac{8}{3} - 2\right) - \left(-18 + 9 - \frac{9}{2}\right)$$

$$= 10 - \frac{8}{3} + 9 + \frac{9}{2} = \boxed{19\frac{11}{6} \text{ or } \frac{125}{6} \text{ units}^2}$$