Calc AB

Lesson 66

11/16/15

Agenda: 11/16/15
- Lesson 66
  - U substitution
  - Change of Variables
- ★ HW 66 Due tomorrow

★ Test on Friday (Lessons 1-66)

IDEA: $\int f(g(x))\cdot g'(x)\,dx$ let $u = g(x)$

then $du = g'(x)\cdot dx \Rightarrow dx = \dfrac{du}{g'(x)}$

$= \int f(u)\,du$] Easier to integrate!

Ex. 66.1

$\int 40x(x^2-4)^5\,dx$ $u = x^2-4$

$= \int 40x(u)^5 \dfrac{du}{2x}$ $du = 2x\,dx$

$= \int 20(u)^5\,du$

$= \frac{20}{6}u^6 + C = \boxed{\frac{10}{3}(x^2-4)^6 + C}$

Ex. 66.3

$\int 7x\sqrt{x-1}\,dx$ $u = x-1$

$du = dx$

$= \int 7x(u)^{1/2}\,du$ $x = u+1$

$= \int 7(u+1)u^{1/2}\,du = 7\int(u^{3/2}+u^{1/2})\,du = 7\cdot\frac{2}{5}u^{5/2} + 7\cdot\frac{2}{3}u^{3/2} + C$

$= \frac{14}{5}(x-1)^{5/2} + \frac{14}{3}(x-1)^{3/2} + C$

$= \boxed{\frac{14}{15}(x-1)^{3/2}[3x-3+5] + C}$

Change of Variables:

Ex. 66.5 $\int_0^{\pi} e^{\sin(5x)}\cos(5x)\,dx$

$u = \sin(5x)$ $= \frac{1}{5}\int_0^0 e^u\,du = \boxed{0}$

$du = \cos(5x)\cdot 5dx$

$x=0 \rightarrow u=0$

$x=\pi \rightarrow u=0$

Ex. 66.4 $\int_0^1 x\sin(\pi x^2)\,dx$

$u = \pi x^2$ $= \int_0^{\pi} \dfrac{\sin(u)}{2\pi}\,du = \frac{1}{2\pi}\left[-\cos(u)\right]_0^{\pi}$

$du = 2\pi x\,dx$ $= \frac{1}{2\pi} + \frac{1}{2\pi} = \boxed{\frac{1}{\pi}}$

$x=0 \rightarrow u=0$

$x=1 \rightarrow u=\pi$

Substitution Theorem for Definite Integrals:

- $f(g(x))$ continuous on $[a, b]$
- $f(u)$ continuous on $[u(a), u(b)]$ where $u = g(x)$

$$\Rightarrow \boxed{\int_a^b f(g(x)) \cdot g'(x)\,dx = \int_{u(a)}^{u(b)} f(u)\,du}$$

<u>Proof</u>:

$$\int_a^b f(g(x))\, g'(x)\,dx = \Big[F(g(x))\Big]\Big|_a^b = F(g(b)) - F(g(a)) \checkmark$$

Because $\frac{d}{dx}\left[F(g(x))\right] = f(g(x)) \cdot g'(x)$

$$\int_{u(a)}^{u(b)} f(u)\,du = F(u(b)) - F(u(a))$$

$$= F(g(b)) - F(g(a)) \checkmark$$

$\blacksquare$