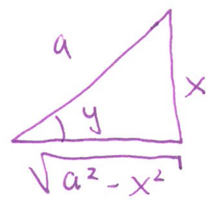
Calc AB — Lesson 64 — 11/9/15

Agenda: 11/9/15
Lesson 64
Derivatives of Inverse Trigonometric functions

- Given $y = \arcsin\left(\frac{x}{a}\right)$ find y'

$\Updownarrow$

$\boxed{\text{Implicit Form}}$ $\sin(y) = \frac{x}{a}$



Ex. $\frac{d}{dx}\left(\sin^{-1}(\cos(x))\right) = \frac{1}{\sqrt{1-\cos^2 x}} \cdot -\sin(x) = \frac{-\sin(x)}{\sin(x)}$

Ex. $\frac{d}{dx}\left(\arctan(2x)\right) = \frac{1}{1+4x^2} \cdot 2 = \frac{2}{1+4x^2}$

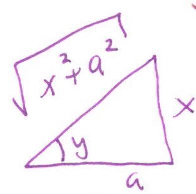
Ex. $\int \frac{1}{1+x^2}\,dx = \tan^{-1}\left(\frac{x}{1}\right) + C$

$$\frac{d}{dx}(\sin(y)) = \frac{d}{dx}\left(\frac{x}{a}\right) \Rightarrow \cos(y) \cdot \frac{dy}{dx} = \frac{1}{a} \Rightarrow \frac{dy}{dx} = \frac{1}{a\cos(y)} = \frac{1}{a}\sec(y)$$

$$\boxed{\frac{dy}{dx} = \frac{1}{a}\left(\frac{a}{\sqrt{a^2 - x^2}}\right) = \frac{1}{\sqrt{a^2 - x^2}}} \qquad \boxed{\frac{d}{dx}\left(\arccos\left(\frac{x}{a}\right)\right) = \frac{-1}{\sqrt{a^2 - x^2}}}$$

Interchange the sides of the triangle

Given $y = \arctan\left(\frac{x}{a}\right)$ find $\frac{dy}{dx}$



$$\frac{d}{dx}(\tan(y)) = \frac{d}{dx}\left(\frac{x}{a}\right) \Rightarrow \sec^2(y)\frac{dy}{dx} = \frac{1}{a} \Rightarrow \frac{dy}{dx} = \cos^2(y) \cdot \frac{1}{a}$$

$\cancel{\frac{dy}{dx} = \left(\frac{\sqrt{a^2 - x^2}}{a}\right)^2}$

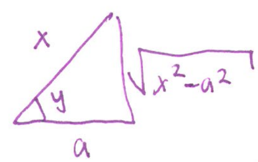
$$\boxed{\frac{dy}{dx} = \left(\frac{a}{\sqrt{x^2 + a^2}}\right)^2 \cdot \frac{1}{a} = \frac{a}{a^2 + x^2}}$$

Interchange the sides of the triangle

$$\boxed{\frac{d}{dx}\left(\operatorname{arccot}\left(\frac{x}{a}\right)\right) = \frac{-a}{a^2 + x^2}}$$

- Given $\sec^{-1}\left(\frac{x}{a}\right) = y$ find $\frac{dy}{dx}$

$\boxed{★ \text{ for } \frac{x}{a} > 0}$



$$\frac{d}{dx}(\sec y) = \frac{1}{a} \Rightarrow \sec y \tan y \frac{dy}{dx} = \frac{1}{a} \Rightarrow \frac{dy}{dx} = \cos y \cdot \frac{1}{\tan(y)} \cdot \frac{1}{a}$$

$$\boxed{\frac{dy}{dx} = \left(\frac{a}{x}\right) \cdot \left(\frac{a}{\sqrt{x^2 - a^2}}\right) \cdot \frac{1}{a} = \frac{a}{|x|\sqrt{x^2 - a^2}}} \qquad \boxed{\frac{d}{dx}\left(\csc\left(\frac{x}{a}\right)\right) = \frac{-a}{|x|\sqrt{x^2 - a^2}}}$$

Without this

Interchange the sides of the triangle