Calc AB

Lesson 62

11/4/15

Agenda: 11/4/15

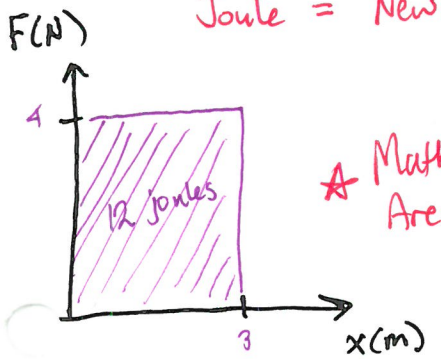
Lesson 62

Work, Distance, and rates

Def When a Uniform force moves an object in the direction of the force, the

Mechanical work done = force x distance traveled

↑ Joule = Newton · Meters ↑ Usually in Newtons ↑ meters



★ Mathematical Area represents the work done

★ Units for any rectangular Area is equal to the product of units for the horizontal and vertical measurements.

$W = \int_a^b F(x)\,dx$

★ Thus if we have a force dependent upon the distance traveled We can define the work done as the area under the force curve!

Ex 62.2 A variable force $F = \frac{1}{2}x^2$ newtons is applied to an object to move it 6 meters in the direction of the force from $x=0$ to $x=6$. What is the work done by the force?

$$\int_0^6 F(x)\,dx = \int_0^6 \frac{1}{2}x^2\,dx = \frac{x^3}{6}\Big|_0^6 = \boxed{36 \text{ joules}}$$

The work done by the force is 36 joules

Ex. A car has a velocity modeled by $v(t) = 50t - 20t^2$ in mph where t is in hours. How far did the car travel between the first and second hour?

$$\int_1^2 (50t - 20t^2)\,dt = \frac{50t^2}{2} - \frac{20t^3}{3}\Big|_1^2 = \left(100 - \frac{160}{3}\right) - \left(25 - \frac{20}{3}\right) = 75 - \frac{140}{3} = \frac{85}{3} \text{ miles}$$

The car traveled $\frac{85}{3}$ miles between the first and second hour.

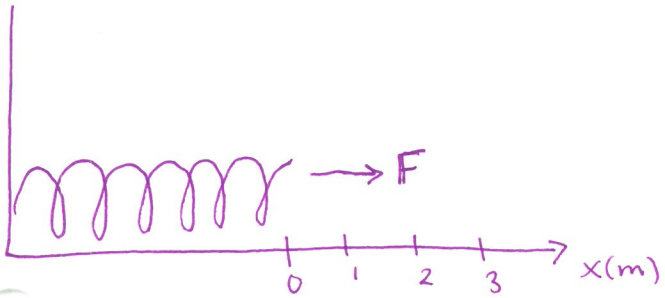
Lesson 62

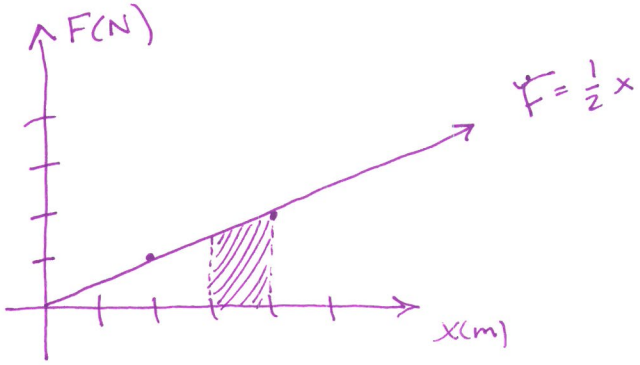
Ex. Hooke's Law for perfectly elastic springs:

The force on a spring is proportional to the Displacement of the spring from the position of rest

$F = Kx$ where K is the <u>spring constant</u> and x represents the displacement of the spring.

Ex 62.4 If K is $\frac{1}{2}$ Newton per meter, how much work is done in stretching the spring from 3 meters to 4 meter?





Work of the spring from 3m to 4m $= \int_3^4 \frac{1}{2}x\,dx = \frac{x^2}{4}\Big|_3^4 = \frac{16}{4} - \frac{9}{4} = \boxed{\frac{7}{4} \text{ joules}}$

Ex. A pool is leaking at a rate of $R(t) = e^{-3t}$ gallons per day. How much water leaked out of the pool after 5 days?

Water after 5 minutes $= \int_0^5 e^{-3t}\,dt = \frac{e^{-3t}}{-3}\Big|_0^5 = \frac{1}{3} - \frac{e^{-15}}{3} \approx 0.333$ gallons