Calc AB

Lesson 57

10/22/15

Agenda: 10/22/15

Period 3

Period 4

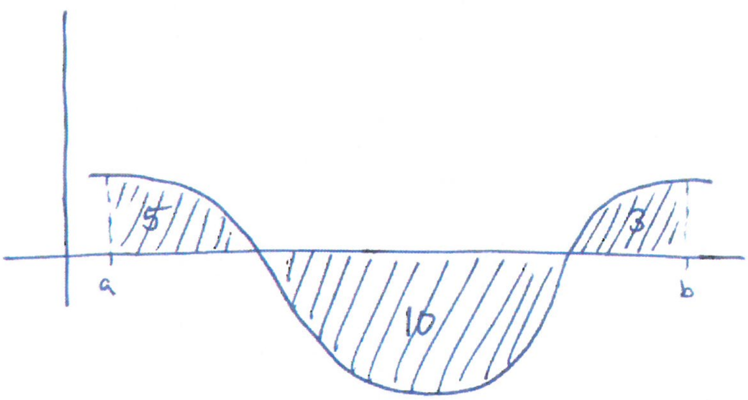
HW Leader:

Lesson 57

Properties of the Definite Integral

★ Quiz 6 tomorrow

- The definite integral is a number that is the limit of a Rieman sum.
- The definite integral is the sum of areas above the x-axis below f and the negatives of the areas above the graph below the x-axis.



$$\int_a^b f(x)\,dx = 5 - 10 + 3 = \boxed{-2}$$

Properties of the Definite Integral

- $\int_a^b (f(x)+g(x))\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$
- $\int_a^b k f(x)\,dx = k\int_a^b f(x)\,dx$
- $\int_b^a f(x)\,dx = F(a) - F(b) = -(F(b) - F(a)) = -\int_a^b f(x)\,dx$
- $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$

$\int_a^a f(x)\,dx = \int_a^b f(x)\,dx + \int_b^a f(x)\,dx = \int_a^b f(x)\,dx - \int_a^b f(x)\,dx = 0$

- If $f(x) \geq 0$ on $[a,b]$, then $\int_a^b f(x)\,dx \geq 0$
- If $f(x) = 0$ on $[a,b]$, then $\int_a^b f(x)\,dx = 0$
- If $f(x) < 0$ on $[a,b]$, then $\int_a^b f(x)\,dx < 0$

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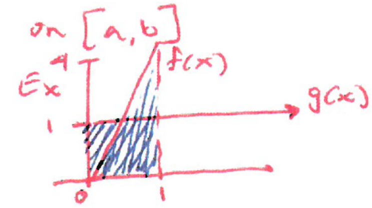
- If $g(x) < f(x)$ on $[a,b]$ then $\int_a^b g(x)\,dx < \int_a^b f(x)\,dx$

2 $\int_a^b g(x)\,dx < \int_a^b f(x)\,dx$ does not mean $g(x) < f(x)$ on $[a,b]$

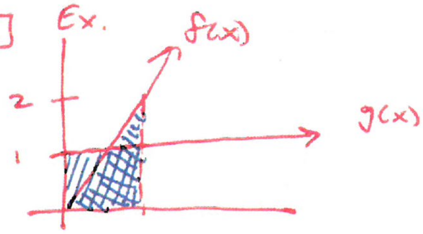
Ex



- If $g(x) = f(x)$ on $[a,b]$ then $\int_a^b g(x)\,dx = \int_a^b f(x)\,dx$

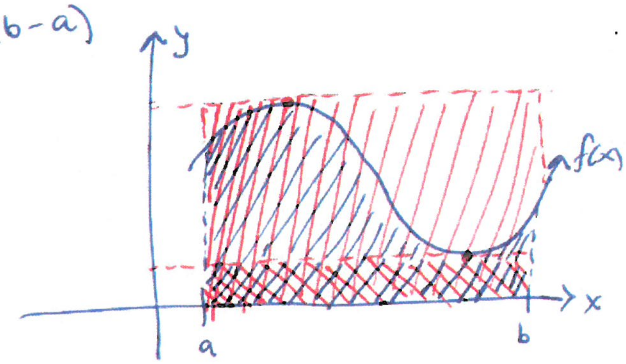
2 $\int_a^b g(x)\,dx = \int_a^b f(x)\,dx \not\Rightarrow g(x) = f(x)$ on $[a,b]$

Ex.



Let $M =$ max of f on $[a,b]$ $m =$ min of f on $[a,b]$

$$m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a)$$



Ex

$\int_{-1}^{1} f(x)\,dx = 7 \quad \int_1^4 f(x)\,dx = 2$

Find $\int_4^{-1} f(x)\,dx$

$$\int_4^{-1} f(x)\,dx = -\int_{-1}^{4} f(x)\,dx = -\left[\int_{-1}^{1} f(x)\,dx + \int_1^4 f(x)\,dx\right] = \boxed{-9}$$

Try Ex. 57.3 $\int_{-2}^{1} f(x)\,dx = 3 \quad \int_3^1 f(x)\,dx = 7$ Find $\int_{-2}^{3} f(x)\,dx$

$$= -7 + 3 = \boxed{-4}$$