Calc AB

Lesson 52+53

10/19/15

Agenda: 10/19/15

HW leader:

lessons 52+53

Optimization Problems

Numerical integration

- Handout Calendar

★ Test 3 back after lesson

Critical numbers: local min, local max, inflection pts or where the derivative is undefined

When the derivative is 0 (horizontal tangent line to curve) or undefined

Optimization problems are applied problems that ask for the absolute (global) minimum or maximum of a function on an interval.

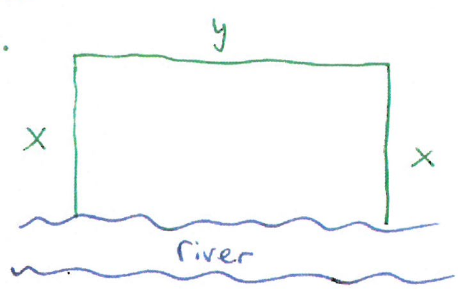
1. Finding the absolute max (or min) on an interval starts by finding all critical numbers of a function on the interval [This includes the endpoints]
2. Then find the function values at the critical numbers.
3. Choose the greatest (or least) value as answer.

Ex. 52.1 A man with 100 yards of fence wants to form a rectangular field enclosed on 3 sides by fence and one side by a river. Find the greatest area that the fence can enclose.

to maximize

1. P = Perimeter   A = Area

1.



2. $P = 100 = y + 2x$ so $y = 100 - 2x$

$A = x \cdot y$ or $A = x(100 - 2x)$ Domain: $(0, \infty)$

3. Find global max of A

$\frac{dA}{dx} = 100 - 4x$ $\frac{dA}{dx} = 0$ when $\boxed{x = 25}$ only critical number

4. Check this is a max of A:

$\frac{d^2A}{dx^2} = -4$ so $\left.\frac{d^2A}{dx^2}\right|_{x=25} = -4 < 0 \Rightarrow$ $\boxed{\text{Local Max of A}}$

Since only critical number this is the global max.

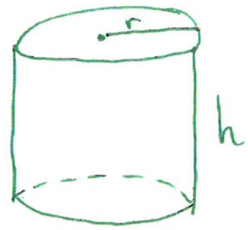
Calc AB $\boxed{\text{lesson } 52 + 53}$ 10/19/15

Ex. A cylindrical can is to be made to hold 16 in^3. If the material for the top and bottom costs \$0.03 per in^2 and the material for the side costs \$0.02 per in^2, find the dimensions which minimize the cost if the height must be between 1 and 5 inches.

1. Draw a picture label variables

   V = volume = 16 in^3

   C = Cost of Can



2. $V = 16 = \pi r^2 \cdot h$ so $h = \frac{16}{\pi r^2}$

   $C = 0.03\,(2\pi r^2) + 0.02\,(2\pi r h)$ so $C = 0.06\pi r^2 + 0.04\pi r\left(\frac{16}{\pi r^2}\right)$

3. Minimize $C(r)$ on $\left[\frac{16}{25\pi}, \frac{16}{\pi}\right]$ $C = .06\pi r^2 + \frac{.64}{r}$ Domain h : $[1,5]$

   r : $\left[\frac{16}{25\pi}, \frac{16}{\pi}\right]$

   Critical numbers: $\frac{dC}{dr} = .12\pi r - \frac{.64}{r^2} = 0$

   $.12\pi r^3 = .64$

   $r^3 = \frac{16}{3\pi}$ $r = \left(\frac{16}{3\pi}\right)^{1/3}$, $\frac{16}{25\pi}$, $\frac{16}{\pi}$

4. Check for global minimum:

   $C\left(\left(\frac{16}{3\pi}\right)^{1/3}\right) \approx \$0.804739$ ← Global minimum when $\boxed{r = \left(\frac{16}{3\pi}\right)^{1/3} \text{ inches} \quad h = \frac{16^{1/3} \cdot 3^{2/3}}{\pi^{1/3}} = \left(\frac{16 \cdot 9}{\pi}\right)^{1/3} \text{ inches}}$

   $C\left(\frac{16}{25\pi}\right) \approx \$3.14942$

   $C\left(\frac{16}{\pi}\right) \approx \$5.0149$

Lesson 53: Numerical Integration of Positive-Valued Functions on a Graphing Calc.

x. 53.2: Use a graphing calculator to approximate the area under the curve $y = \sin(x)$ between $x = 0$ and $x = \pi/6$.

$$\text{Area} = \int_0^{\pi/6} \sin(x)\,dx = -\cos(x)\Big|_0^{\pi/6} = -\cos\left(\tfrac{\pi}{6}\right) + \cos(0) = 1 - \tfrac{\sqrt{3}}{2} \approx 0.133976$$

$$\approx \underbrace{\text{fnInt}}_{\text{Under } \boxed{\text{MATH}}}(\overset{\text{function}}{\sin(x)}, \underbrace{0}_{\text{start}}, \overset{\text{end}}{\pi/6}) \approx 0.133975$$

★ Calc in RADIAN Mode!