Calc AB

Lesson 49

10/5/15

Agenda: 10/5/15

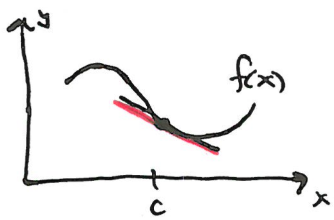
HW leader: None

Lesson 49:

First and Second derivative Tests

$f(x)$ a continuous function with a derivative at $x = c$

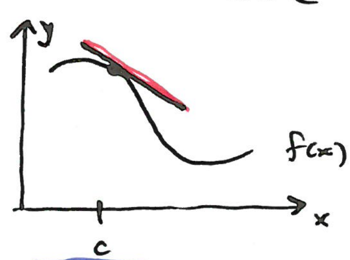
1\.



Concave up

All points near c above tangent line

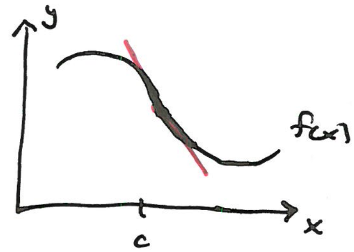
2\.



Concave down

All points near c below tangent line

3\.



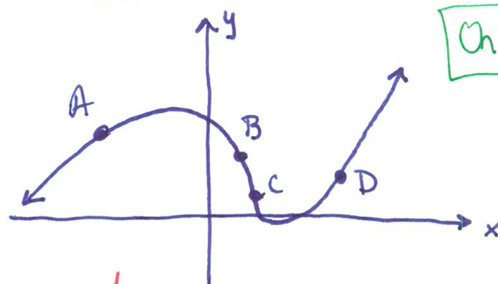
Inflection Point

All points near on one side above and all points near on the other ~~point~~ side below

f	↑ / ↓	$\cup$ / $\cap$
f'	+ / −	↑ / ↓
f''		+ / −

Ex. 49.2

Where is $\frac{d^2y}{dx^2}$ positive?



Only C and D

First Derivative Test: $f'(c) = 0$

- $(c, f(c))$ is a local max if $\quad + \;\big|_{c}\; -\quad$ sign of f'
- $(c, f(c))$ is a local min if $\quad - \;\big|_{c}\; +\quad$ sign of f'
- $(c, f(c))$ is an inflection point if no change in sign of f'.

Second Derivative Test: $f'(c) = 0$

- If $f''(c) > 0$ then $(c, f(c))$ is a local minimum.
- If $f''(c) < 0$ then $(c, f(c))$ is a local maximum.

☆ If $f''(c) = 0$ then we **must** use the first Derivative test.

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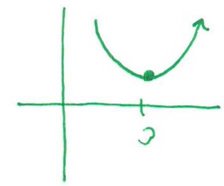
r. 49.3 Suppose f is a polynomial function such that

$$f'(3) = 0 \quad \text{and} \quad f''(3) = 3.$$

Sketch a possible graph of f near 3 and indicate the property of the function f at $x = 3$.

$f''(3) > 0 \Rightarrow$ concave up $\Rightarrow$ local min at $x = 3$



Ex. Without using a calculator find all local min, max and inflection points for

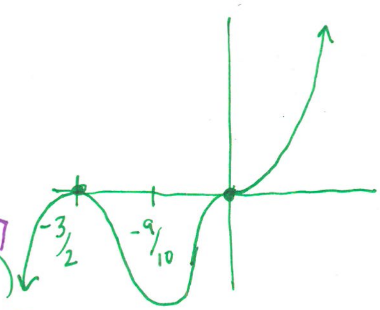
$$f(x) = \underbrace{x^3}\underbrace{(2x+3)^2}$$

$$f'(x) = 3x^2(2x+3)^2 + x^3(2x+3)\cdot 4 = x^2(2x+3)\left[6x+9+4x\right]$$

$$= x^2(2x+3)(10x+9)$$

$$= x^2(20x^2+48x+27)$$

Critical Numbers: $x = 0,\ x = -\frac{3}{2},\ x = -\frac{9}{10}$



$$f''(x) = 2x(2x+3)(10x+9) + \overbrace{2x^2(10x+9) + 10x^2(2x+3)}$$

$$= 2x\left[20x^2 + 18x + 30x + 27 + 10x^2 + 9x + 10x^2 + 15x\right]$$

$$= 2x\left[40x^2 + 72x + 27\right]$$

$$f''(x) = 2x\left[20x^2 + 48x + 27 + 20x^2 + 24x\right]$$
$$= 2x\left[40x^2 + 72x + 27\right]$$

$$f''(-\tfrac{3}{2}) = (-3)\left[90 - 36\cdot 3 + 27\right] \not> 0$$

$$f''(-\tfrac{9}{10}) = \left(-\tfrac{9}{5}\right)\left[36 - (7.2)\cdot 9 + 27\right] > 0$$

} Local maxima, local minimum by 2nd Derivative test

First derivative test for $x = 0$:

Sign of f': $+$ $+$ ($-\frac{1}{2}$, 0, $\frac{1}{2}$) $\Rightarrow$ inflection point

Local max: $\left(-\frac{3}{2}, 0\right)$

Local min: $\left(-\frac{9}{10}, -\frac{6561}{6250}\right)$

Inflection point: $(0, 0)$