Calc AB

Lesson 47

10/1/15

Agenda: 10/1/15

Period 3

Period 4

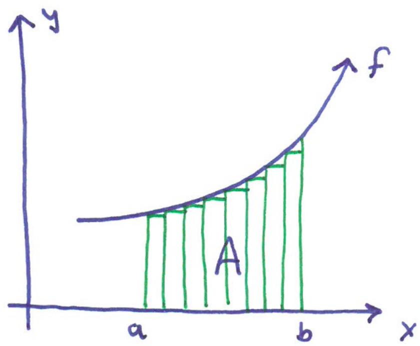
HW leader :

lesson 47

Fundamental Theorem of Calculus

Definite Integral

Quiz 5 tomorrow



Definition: For f a continuous non-negative function

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x \quad \text{where} \quad \Delta x = \frac{b-a}{n}$$

We denote by $A = \int_a^b \underbrace{f(x)}_{\text{height}}\,\underbrace{dx}_{\text{very small width}}$ ← sum of infinitely rect. from a to b.

Called a **Definite Integral**

a - lower bound

b - upper bound

Fundamental Theorem of Calculus

If f is continuous on $[a,b]$ and $F(x)$ is any antiderivative of $f(x)$ then

$$\boxed{\int_a^b f(x)\,dx = F(b) - F(a)}$$

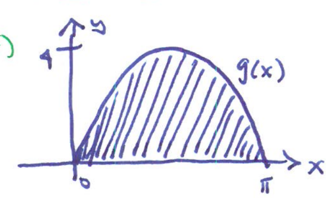
Much faster than using Limits!

Ex. 47.1 Find the area under the graph of $g(x) = 4\sin(x)$ on $[0, \pi]$

$$A = \int_0^{\pi} 4\sin(x)\,dx = -4\cos(x)\Big|_0^{\pi} = -4\cos(\pi) + 4\cos(0) = \boxed{8 \text{ units}^2}$$

$G(x) = -4\cos(x)$ is an antiderivative of $g(x)$

because $\frac{d}{dx}G(x) = 4\sin(x) = g(x)$.

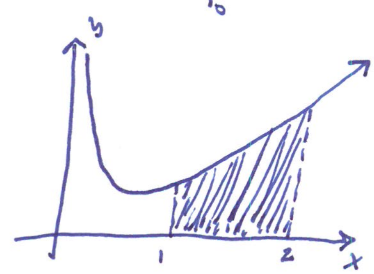


Ex. 47.3 Evaluate: $\int_1^2 \left(3e^x + \frac{2}{x} + x^2\right) dx$

$$= \left(3e^x + 2\ln|x| + \frac{x^3}{3}\right)\Big|_1^2$$

$$= 3e^2 + 2\ln(2) + \frac{8}{3} - 3e - 2\ln(1) - \frac{1}{3}$$

$$= \boxed{3e^2 + 2\ln(2) + \frac{7}{3} - 3e} \approx 17.732$$



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The Definite Integral: $\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\cdot\Delta x$

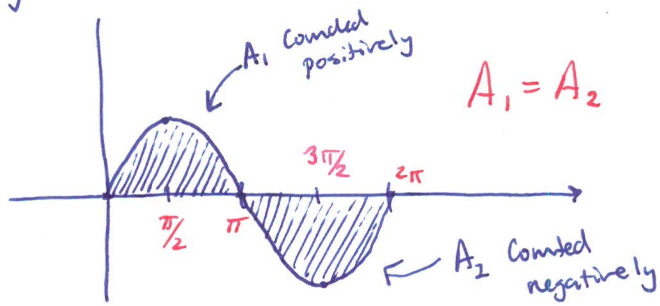
- Definite integral is a NUMBER whereas the Indefinite Integral is a FAMILY of functions
- Limit of a sum of products involving function values

Thus it's still applicable for functions with negative values BUT the value doesn't represent the area under a curve.

If f is negative then $\int_a^b f(x)\,dx$ is negative so $-\int_a^b f(x)\,dx$ represents the area between the x-axis and the graph of f.

Ex. 47.5 Find $\int_0^{2\pi} \sin(x)\,dx$ geometrically

$= A_1 - A_2 = \boxed{0}$



$A_1 = A_2$

Your Turn

HW # 6

- Find the area under the graph of $y = x^{1/3}$ on $[1, 3]$

$$\int_1^3 x^{1/3}\,dx = \left[\frac{3x^{4/3}}{4}\right]\Bigg|_1^3$$

$$= \frac{3(3)^{4/3}}{4} - \frac{3}{4}$$

$$= \frac{3^{7/3} - 3}{4}$$

$$= \boxed{\frac{9\sqrt[3]{3} - 3}{4}} \approx \boxed{2.495}$$