Calc AB

Lesson 42 | +43

9/24/15

Agenda: 9/24/15 — Period 3 — Period 4

HW leader:

Lesson 42 + 43

Quotient Rule

Area under a curve as an Infinite Sum

Quotient Rule:

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{d}{dx}\left(f(x)\cdot g(x)^{-1}\right)$$

$$= f'(x)\cdot g(x)^{-1} + f(x)\cdot \frac{d}{dx}\left(g(x)^{-1}\right)$$

$$= f'(x)\cdot g(x)^{-1} - f(x)\cdot g(x)^{-2}\cdot g'(x)$$

$$\boxed{= \frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{[g(x)]^2}}$$

$$\boxed{d\left(\frac{u}{v}\right) = \frac{v\,du - u\,dv}{v^2}}$$

Ex 42.1 $f(x) = \dfrac{\ln(x)}{2x+3}$, find $f'(x)$.

$$f'(x) = \frac{(2x+3)\frac{1}{x} - \ln(x)\cdot(2)}{(2x+3)^2} = \boxed{\frac{2x+3-2x\ln(x)}{x(2x+3)^2}}$$

Ex. Find dw where $w = \dfrac{\cos(z)}{e^z + z^2}$

$$\boxed{dw = \frac{-(e^z+z^2)\sin(z)dz - \cos(z)(e^z+2z)dz}{(e^z+z^2)^2}}$$

Your turn: Find $\frac{d}{dx}(\csc(x))$ by writing $\csc(x)$ in terms of sine and cosine

$$= \frac{d}{dx}\left(\frac{1}{\sin(x)}\right) = \frac{0 - 1(-\cos(x))}{\sin^2(x)} = \boxed{\cot(x)\cdot\csc(x)}$$

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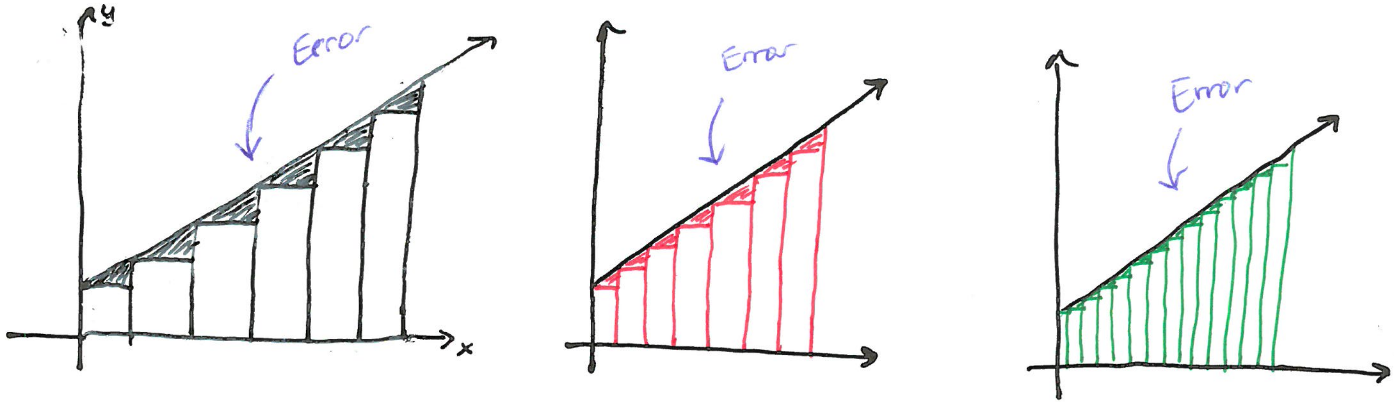


Error gets smaller and smaller as we get more and more rectangles in our interval.

Define the area on $[a,b]$ between the curve f and the x-axis to be A where

$$A = \lim_{n\to\infty} S_L(n) = \lim_{n\to\infty} S_U(n)$$

$$S_L(n) = \sum_{i=1}^{n} \Delta x\, f(x_{L_i}) \qquad S_U(n) = \sum_{i=1}^{n} \Delta x\, f(x_{U_i})$$

Ex. 43.1 Use inscribed rectangles to find the exact area under $g(x) = 2x$ on $[0,2]$ by allowing the number of rectangles to increase without bound.

$$S_L(n) = \sum_{i=0}^{n-1} \Delta x\, g(x_i) \qquad \Delta x = \frac{2-0}{n} = \frac{2}{n}$$

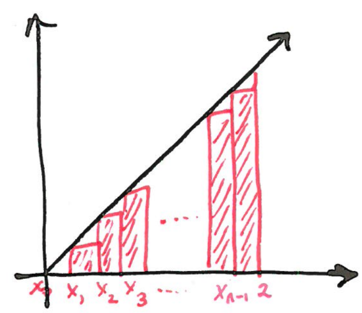
$$= \sum_{i=0}^{n-1} \frac{2}{n}\,\frac{4i}{n}$$

$$= \frac{8}{n^2}\left[\sum_{i=0}^{n-1} i\right]$$

$$= \frac{8}{n^2}\,\frac{(n-1)(n)}{2}$$

$$= \frac{4(n-1)}{n}$$

i	x	g(x)
0	0	0
1	$\frac{2}{n}$	$\frac{4}{n}$
2	$\frac{4}{n}$	$\frac{8}{n}$
3	$\frac{6}{n}$	$12/n$
n-1	$\frac{2(n-1)}{n}$	$\frac{4(n-1)}{n}$



Recall:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$A = \lim_{n\to\infty} S_L(n) = \lim_{n\to\infty} \frac{4(n-1)}{n} = 4$$