Calc AB

Lesson 40

9/17/15

Agenda: 9/17/15

HW Leader:

Lesson 40

Units for the Derivative

Normal lines

A Test 3 tomorrow

Period 3	Period 4
Chris C.	Katie O.

Q: What is the derivative of a function?

A: Function for the slope of the tangent line to the function at a point

$$\text{Slope} = \frac{\Delta y}{\Delta x}$$ so the units for slope and hence the derivative are

$$\frac{\text{Units of dependent variable}}{\text{Units of independent variable}}$$

Ex. 40.2 The velocity of a particle in m/s is given by $v = 4t^2 + 2t + 4$ where t is in seconds. What is the acceleration of the particle when $t = 3$ seconds?

Acceleration is the rate of change of velocity wrt time.

$$\text{Acceleration} = \frac{dv}{dt} = (8t + 2)\ \frac{m/s}{s} = (8t+2)\ m/s^2 \checkmark$$

$$\left.\frac{dv}{dt}\right|_3 = 8(3) + 2 = 26\ m/s^2$$

At 3 seconds the particles acceleration is 26 m/s^2.

Position $x(t)$ —ROC→ velocity $v(t) = \frac{dx(t)}{dt}$ —ROC→ acceleration $a(t) = \frac{dv(t)}{dt} = \frac{d^2x(t)}{dt^2}$ —ROC→ jerk $j(t) = \frac{da(t)}{dt} = \frac{d^2v(t)}{dt^2} = \frac{d^3x(t)}{dt^3}$

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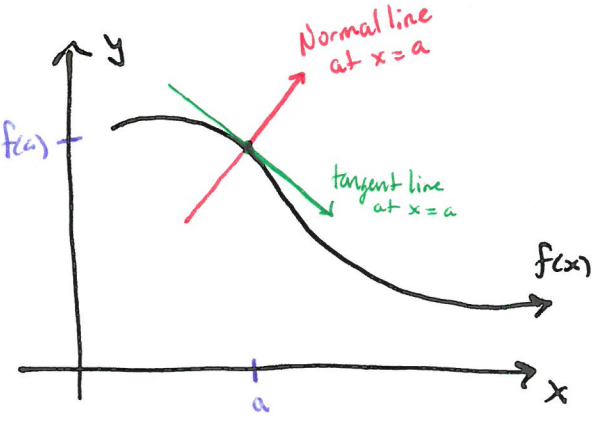
Normal lines:

★ Normal means perpendicular



Recall: Tangent line to f at $x = a$

$$y = f'(a)(x - a) + f(a)$$

Slope of perpendicular lines: negative reciprocal

Normal line to f at $x = a$:

$$y = -\frac{1}{f'(a)}(x - a) + f(a)$$

∴ Find the equation of the line normal to $f(x) = 3\sin(x)$ at $x = \pi/6$

Point: $(\pi/6, f(\pi/6)) = (\pi/6, 3/2)$

Slope: $-\frac{1}{f'(\pi/6)}$ $\quad f'(x) = 3\cos(x)$

$$-\frac{1}{f'(\pi/6)} = \frac{-1}{3\cos(\pi/6)} = \frac{-2}{3\sqrt{3}} = -\frac{2\sqrt{3}}{9}$$

Normal line at $x = \pi/6$:

$$y = -\frac{2\sqrt{3}}{9}(x - \pi/6) + \frac{3}{2}$$

Ex: [Students]

- Find the normal line to $f(x) = \ln(x^2)$ at $x = 1$

  $y = -\frac{1}{2}(x - 1) + \ln(1)$

- The cost of operating a school is given by $C(x) = \$500x + \$2500$ where x is the number of students. What are the units of $C'(x)$? $ per student