Calc AB

Lesson 39

9/21/15

Agenda: 9/21/15

HW leader: Period 3 Jared G. Period 4 Jonie P.

lesson 39

Area under a Curve

Upper/Lower Sums

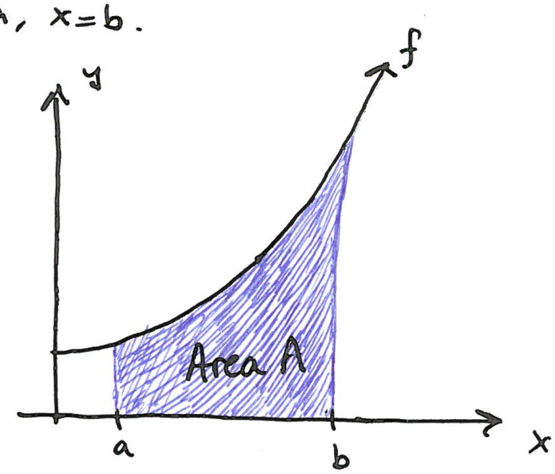
Test back after lesson

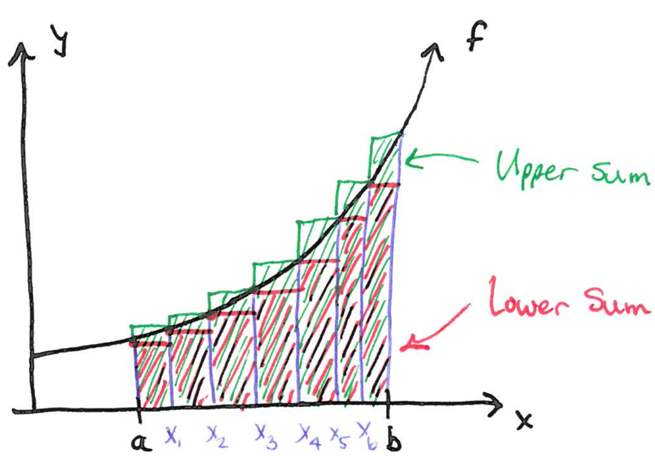
In mathematics, areas can be used to represent distance, work, total force etc.

★ Area is a real number used to numerically describe an abstract quality associated with evey closed planar figure.

Definition - Given a continuous non-negative function f on the interval $[a,b]$ We define the area under the curve f on $[a,b]$ to be the area bounded by the graph of f, the x-axis and the lines $x=a$, $x=b$.

We begin by trying to approximate this area under the curve by using rectangles to partition it.





Definition - the Lower Sum, S_L, is the sum of the areas of the rectangles of the partition of $[a,b]$ with the least value of $f(x)$ choosen as the height on each sub interval.

★ If f is increasing then the choosen height for Lowersums on each subinterval is on the left, f decreasing then on the right.

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Definition – the <u>upper sum</u>, S_U, is the sum of the areas of the rectangles of the partition of $[a,b]$ with the largest value of $f(x)$ choosen as the height on each sub interval.

★ If f is increasing, choosen height on right. f decreasing then on the left.

★ Both S_U and S_L are approximations to the actual area A.

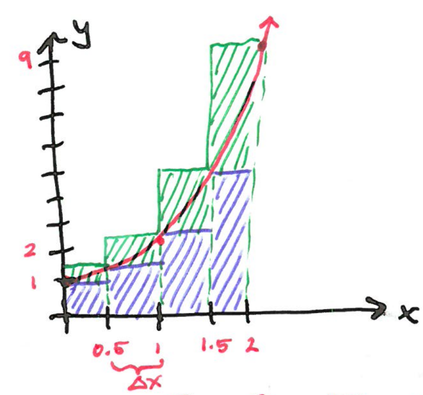
Under estimate → $\boxed{S_L \le A \le S_U}$ ← overestimate

★ Given the partition of $[a,b]$, though S_L and S_U are estimates, everyone doing the problem will arrive at the same answer (if done correctly)

Ex. Use both lower and upper sums to estimate the area under

$$f(x) = x^3 + 1$$

on the interval $[0,2]$ divided into $n=4$ equal subintervals by averaging the two.



Partition: $[0,0.5], [0.5,1], [1,1.5], [1.5,2]$

$\Delta x = 0.5 = \frac{2-0}{4}$

$S_U = \frac{1}{2}f(0.5) + \frac{1}{2}f(1) + \frac{1}{2}f(1.5) + \frac{1}{2}f(2)$

$= \frac{1}{2}\left(\frac{9}{8}\right) + \frac{1}{2}(2) + \frac{1}{2}\left(\frac{35}{8}\right) + \frac{1}{2}(9)$

$= \frac{9+16+35+72}{16} = \frac{132}{16} = \boxed{\frac{33}{4}}$

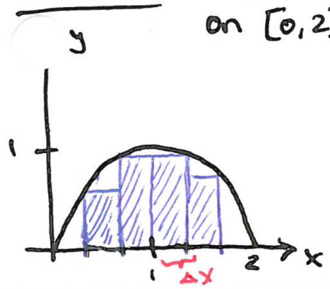
$S_L = \frac{1}{2}f(0) + \frac{1}{2}f(0.5) + \frac{1}{2}f(1) + \frac{1}{2}f(1.5)$

$= \frac{1}{2}(1) + \frac{1}{2}\left(\frac{9}{8}\right) + \frac{1}{2}(2) + \frac{1}{2}\left(\frac{35}{8}\right)$

$= \frac{8+9+16+35}{16} = \frac{68}{16} = \boxed{\frac{17}{4}}$

Average $= \frac{S_U + S_L}{2} = \frac{50}{4} = \boxed{\frac{25}{2}}$

Ex. 39.3 Use a lower sum with $n=6$ subintervals to estimate the area under $y = -x^2+2x$ on $[0,2]$.



$\Delta x = \frac{2-0}{6} = \frac{1}{3}$

$S_L = \frac{1}{3}\cancel{f(0)}^{\,0} + \frac{1}{3}f\left(\frac{1}{3}\right) + \frac{1}{3}f\left(\frac{2}{3}\right) + \frac{1}{3}f\left(\frac{4}{3}\right) + \frac{1}{3}f\left(\frac{5}{3}\right) + \cancel{\frac{1}{3}f(2)}^{\,0}$

$= \frac{1}{3}\left(0 + \frac{5}{9} + \frac{8}{9} + \frac{8}{9} + \frac{5}{9} + 0\right) = \boxed{\frac{26}{27}}$

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HW Leader: None

Lesson 39 Continued

Left, Right, Midpoint Sums

<u>Left Sums</u> – use the left-hand endpoint on each interval : S_{Left}

<u>Right Sums</u> – use the right-hand endpoint on each interval : S_R

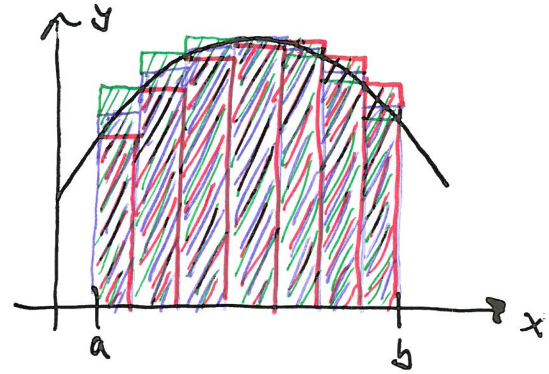
<u>Midpoint Sums</u> – use the midpoint of each interval : S_M

- If f is <u>increasing</u>:

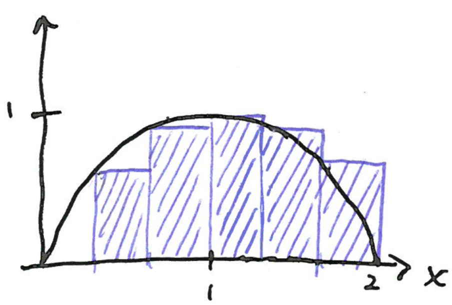
  $S_L = S_{Left}$, $S_U = S_R$

- If f is <u>decreasing</u>:

  $S_L = S_R$, $S_U = S_{Left}$.



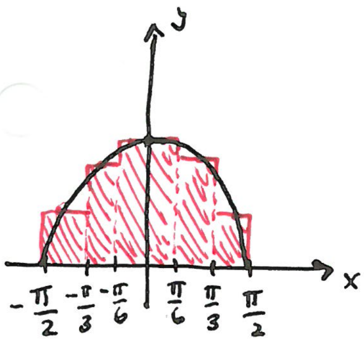
<u>Ex. 39.5</u> Estimate the area under $y = -x^2 + 2x$ on $[0, 2]$ using a left sum with $n = 6$ subintervals.



$$S_{Left} = \frac{1}{3}\left[f(0) + f(\tfrac{1}{3}) + f(\tfrac{2}{3}) + f(1) + f(\tfrac{4}{3}) + f(\tfrac{5}{3})\right]$$

$$= \frac{1}{3}\left[0 + \frac{5}{9} + \frac{8}{9} + 1 + \frac{8}{9} + \frac{5}{9}\right] = \boxed{\frac{35}{27}}$$

<u>Ex.</u> Use midpoint rectangles with $n=6$ to estimate the area under $g(x) = \cos(x)$ on $[-\frac{\pi}{2}, \frac{\pi}{2}]$



$$A^{(g)}_{[-\frac{\pi}{2},\frac{\pi}{2}]} = 2\, A^{(g)}_{[0,\frac{\pi}{2}]} \qquad \Delta x = \frac{\pi}{6}$$

$$= 2\left(\frac{\pi}{6}\right)\left[g\left(\frac{\pi}{12}\right) + g\left(\frac{3\pi}{12}\right) + g\left(\frac{5\pi}{12}\right)\right]$$

$$= \frac{\pi}{3}\left[\cos\left(\frac{\pi}{12}\right) + \cos\left(\frac{\pi}{4}\right) + \cos\left(\frac{5\pi}{12}\right)\right]$$

$$= \frac{\pi}{3}\left[\frac{\sqrt{2+\sqrt{3}} + \sqrt{2} + \sqrt{2-\sqrt{3}}}{2}\right] \approx 2.023$$

$$\cos\left(\frac{\pi}{12}\right) = \sqrt{\frac{1+\cos(\pi/6)}{2}} = \sqrt{\frac{1+\sqrt{3}/2}{2}} = \frac{\sqrt{2+\sqrt{3}}}{2}$$

$$\cos\left(\frac{5\pi}{12}\right) = \sqrt{\frac{1-\sqrt{3}/2}{2}} = \frac{\sqrt{2-\sqrt{3}}}{2}$$