Calc AB

Lesson 34

9/9/15

Agenda: 9/9/15

HW leader:

lesson 34

Implicit Differentiation

Period 3	Period 4
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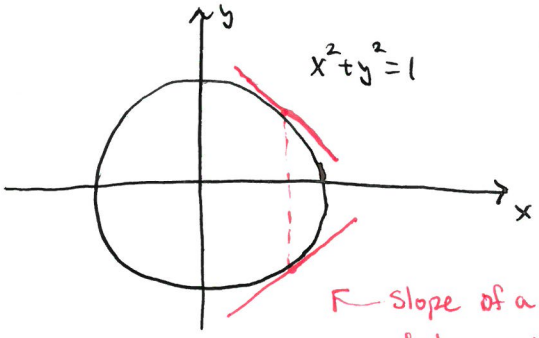
★ Quiz 4 on Friday

Explicit vs Implicit Equations

Explicit equations: form "y equals" Ex. $y = 2x+5$ $y = \frac{2x^2+x}{5x^3-x+1}$

Implicit equations: Anything else Ex $xy+1 = 2x-y$ $x^2+y^2=1$

The method for finding $\frac{dy}{dx}$ is called <u>implicit differentiation</u> when y is defined implicitly.



Though not a function we can still talk of tangent lines to the circle.

← slope of a tangent line is now determined by both x and y.

Implicit Differentiation:

1. Find the differential dy
2. Divide the result by dx

Ex. Find $\frac{dy}{dx}$ where $y^2+x^2=1$

$$2y\,dy + 2x\,dx = 0 \Rightarrow dy = -\frac{x\,dx}{y}$$

$$\boxed{\frac{dy}{dx} = -\frac{x}{y}}$$

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Ex. If $w^2z - we^z = \sin(w)$ find $\frac{dz}{dw}$.

$$2wz\,dw + w^2dz - e^z dw - we^z dz = \cos(w)\,dw$$

$$dz(w^2 - we^z) = (\cos(w) + e^z - 2wz)\,dw$$

$$\frac{dz}{dw} = \frac{\cos(w) + e^z - 2wz}{w^2 - we^z}$$

Ex. Given that x and y are functions of time and $x^2 - 5x + y^2 - 3y - 7 = 0$ find $\frac{dx}{dt}$ at $(1,1)$ when $\frac{dy}{dt} = 2$.

$$2x\,dx - 5dx + 2y\,dy - 3dy = 0$$

$$\frac{dx}{dt} = \frac{3 - 2y}{2x - 5} \cdot \frac{dy}{dt}$$

$$\left.\frac{dx}{dt}\right|_{(1,1)} = \frac{1}{-3}(2) = \boxed{-\frac{2}{3}}$$

Ex. Find the points on the graph of $x^2 + 2x + y^2 - 8 = 0$ where

(a) the tangent is horizontal

$\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = -\frac{(2x+2)}{2y}$$

(a) $0 = \frac{dy}{dx}$ when $2x + 2 = 0$

$\boxed{x = -1}$

When $x = -1$:

$y^2 = 9$ so $y = \pm 3$

$\boxed{(-1, -3) \text{ and } (-1, 3)}$

(b) the tangent is vertical

$\frac{dy}{dx}$ is undefined or $\frac{dx}{dy} = 0$

(b) $\frac{dy}{dx}$ is undefined when $y = 0$

$x^2 + 2x - 8 = 0$

$(x + 4)(x - 2) = 0$

$x = -4 \quad x = 2$

$\boxed{(-4, 0) \text{ and } (2, 0)}$