Calc AB

Lesson 29+31

9/2/15

Agenda: 9/2/15

HW Leader:

Period 3: Ivan M.

Period 4: Chris M.

Lesson 29+31

- Differentials
- Product Rule

★ Quiz 3 on Friday

Second Derivative and Concavity

f	↑↓	∪ ∩
f'	+/−	↑↓
f''		+/−

Notation: Newton Vs. Leibniz

$$\frac{dy}{dx} = \frac{d}{dx}f(x) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

Better notation for practical problems

Newton: $\dot{y} \Rightarrow y'$ Not prefered by most Scientists

Def - Let $y=f(x)$ be a differentiable function.

The <u>differential of x</u> (denoted dx) is any nonzero real number.

The <u>differential of y</u> (denoted dy) is $dy = f'(x)\,dx$.

Ex. 29.2 Let $y = \sin t + \cos t$. Find dy

- y is a function of t Say $f(t) = \sin t + \cos t$ then

$$dy = f'(t)\,dt = \boxed{(\cos t - \sin t)\,dt}$$

Product Rule: f and g are two differentiable functions then

$$\frac{d}{dx}\left(f(x)\cdot g(x)\right) = \left(\frac{d}{dx}f(x)\right)\cdot g(x) + f(x)\cdot\left(\frac{d}{dx}g(x)\right)$$

$$= f'(x)\cdot g(x) + f(x)\cdot g'(x)$$

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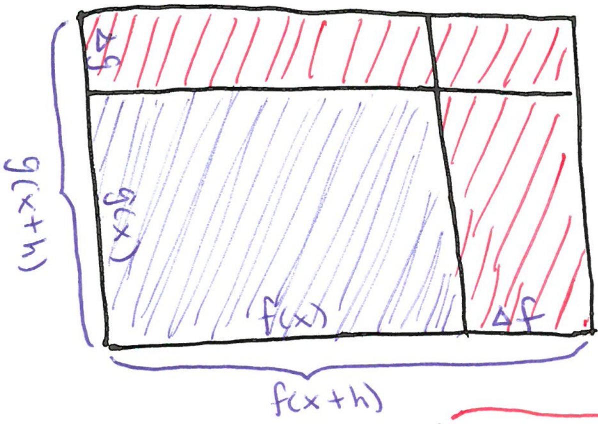
Proof:



Let

$\Delta f = f(x+h) - f(x)$

$\Delta g = g(x+h) - g(x)$

$\longrightarrow$ Same as the red area

$$\frac{d}{dx}\left(g(x)\cdot f(x)\right) = \lim_{h\to 0} \frac{\overbrace{f(x+h)\,g(x+h) - f(x)\,g(x)}}{h}$$

$$= \lim_{h\to 0} \frac{\Delta g\, f(x) + \Delta f\, g(x) + \Delta g\, \Delta f}{h}$$

$$= \lim_{h\to 0} \left(\frac{\Delta g}{h} f(x) + \frac{\Delta f}{h} g(x) + \frac{\Delta g}{h} \Delta f\right)$$

$$= f(x)\cdot g'(x) + f'(x)\,g(x) + g'(x)\cancelto{0}{\left(\lim_{h\to 0} \Delta f\right)}$$

$$= f(x)\cdot g'(x) + f'(x)\,g(x)$$

Ex. 31.4 Let $f(x) = \underbrace{e^x}\,\underbrace{\cos x}$. Find $f'(1.2)$

$f'(x) = e^x \cos x - e^x \sin x$ $\quad f'(1.2) = e^{1.2}\cos(1.2) - e^{1.2}\sin(1.2)$

≈ -1.8914

On TI-84/83

$\boxed{\text{MATH}}\ \boxed{8}$ nDeriv(function, variable, value) nDeriv($e^x \cos(x)$, x, 1.2) ≈ -1.8914

Ex. 31.5 Let $s = x^2 y^3$ find ds.

$d(x^2) = 2x\,dx$ $\quad ds = d(x^2)\,y^3 + x^2\, d(y^3)$

$d(y^3) = 3y^2\,dy$ $\quad \boxed{ds = (2xy^3)\,dx + 3x^2y^3\,dy}$