Calc AB

Lesson 27

8/31/15

Agenda: 8/31/15

HW Leader:

Period 3: Anya K.

Period 4: Vanessa H.

Lesson 27

- Tangent line equations
- Higher order derivatives

★ WS on Representations Due ~~Friday~~ Tuesday 9/8

★ 6 point curve on Test

IF you come in to office hours and explain to me your mistakes and the right answers.

★ Test 1 back at the end

Equation of the Tangent Line

Given a function $f(x)$ that is differentiable the tangent line at $x=a$ is

$$y = f'(a)(x-a) + f(a)$$

[Point-Slope form]

Recall: For a line need a slope and point

$f'(a)$ $(a, f(a))$

Ex. 27.2

Find the equation of the tangent to the graph of $y = x^2 - 4x + 3$ when $x = 3$.

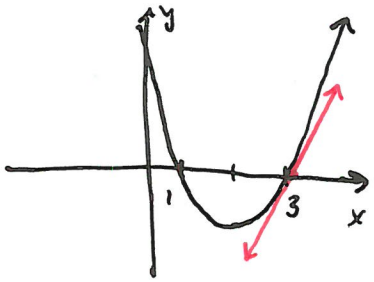
$\frac{dy}{dx} = 2x - 4$

$\frac{dy}{dx}\Big|_3 = 2(3) - 4 = 2$ ← derivative of y evaluated at 3; Slope of tangent line at $x=3$

When $x=3$ $\quad y = (3)^2 - 4(3) + 3 = 0$ $\quad$ Point $(3, 0)$

Tangent Line: $y = 2(x-3) + 0 = \boxed{2x-6}$

Allows us to approximate the function value near $x=3$.



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Ex. 27.3 Find the equation of the line tangent to $y = \sin x$ when $x = \pi/6$.

When $x = \pi/6$, $y = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$ Point: $\left(\frac{\pi}{6}, \frac{1}{2}\right)$

Slope: $\frac{dy}{dx} = \cos(x)$ $\left.\frac{dy}{dx}\right|_{\pi/6} = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

Tangent Line: $\boxed{y = \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right) + \frac{1}{2}}$

Higher Order Derivatives:

- If a function is differentiable then its derivative is another function called the first derivative.
- If the first derivative is differentiable, then its derivative is a function called the second derivative. . . .

Notation

Function: $f(x)$ or y

First Derivative: $f'(x)$ or $\frac{dy}{dx}$

Second Derivative: $f''(x)$ or $\frac{d^2y}{dx^2}$

n^{th} Derivative: $f^{(n)}(x)$ or $\frac{d^n y}{dx^n}$

Slope of tangent line of f at x
Instantaneous Rate of Change of $f(x)$
" " of f' at x
" " of $f'(x)$
⋮

Ex. Let $y = \frac{1}{4}u^3 - \frac{1}{u^2}$. Find $\left.\frac{d^2y}{du^2}\right|_2$

$$\frac{dy}{dx} = \frac{3}{4}u^2 + \frac{2}{u^3} \longrightarrow \frac{d^2y}{du^2} = \frac{3}{2}\cdot u - \frac{6}{u^4}$$

$$\left.\frac{d^2y}{du^2}\right|_2 = 3 - \frac{6}{16} = \frac{42}{16} = \boxed{\frac{21}{8}}$$

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Tangent line equations

Higher order derivatives

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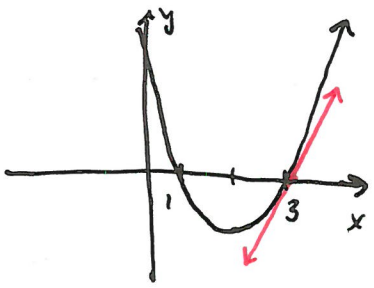

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