Calc AB

Lesson 19

8/20/15

<u>Agenda</u>: 8/20/15

HW Leader:

Period 3: Chris C.

Period 4: Annalisa R.

Lesson 19

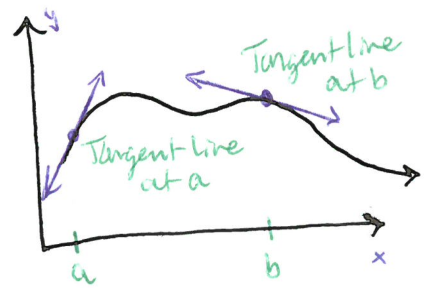
The derivative

★ Quiz 2 Tomorrow

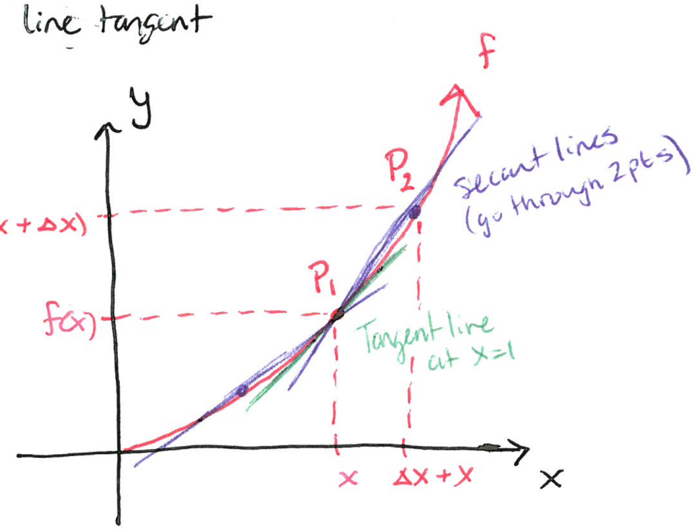
Tangent lines:

A tangent to a curve is a straight line that "touches" the curve.

Slope tells us how steeply a line rises or falls.



★ For nonlinear equations, the slope of a curve at a point is the slope of the line tangent to the curve at that point.



Slope of Secant line:

$$\frac{f(x+\Delta x)-f(x)}{\Delta x + x - x} = \frac{f(x+\Delta x)-f(x)}{\Delta x}$$

As P_2 gets closer to P_1
We get closer and closer to the slope of the tangent line.

<u>Def</u> - The derivative of a function f at a point x is a new function

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x} = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

[Read "f prime"]

Provided the limit exists.

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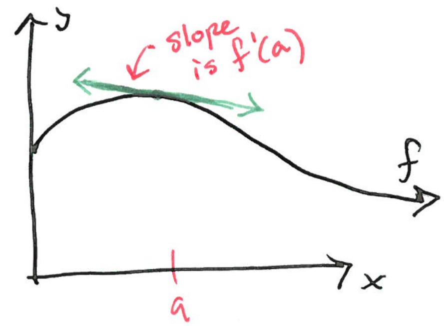
The derivative is a function that gives the slope of the tangent line at a point $\underline{a}$ when the function is given $x=a$ as input.



Notation:

Taking a derivative is an operator that is applied to a function.

[operators] $\boxed{\dfrac{d}{dx} \quad \text{or} \quad D_x}$

[Take Derivative of a function]

$$\frac{d}{dx}(f(x)) \quad \text{or} \quad \frac{df}{dx} \quad \text{or} \quad D_x f(x)$$

$$\frac{dy}{dx} \quad \text{or} \quad \frac{d}{dx} y \quad \text{or} \quad D_x y \qquad \text{if } y = f(x)$$

Ex. 19.1-19.2 Find $\frac{dy}{dx}$ where $y=x^2$ and find the slope of the tangent line at $x=4$.

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h\to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h}$$

$$= \lim_{h\to 0} (2x+h)$$

$$= 2x$$

$$\boxed{\frac{dy}{dx} = 2x} = f'(x) \text{ if } f(x) = x^2$$

Slope of tangent @ $x=4$: $\left.\dfrac{dy}{dx}\right|_{x=4} = 2(4) = 8$

or $f'(4) = 2(4) = 8$.

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Derivative of a Constant: $f(x) = c$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{c-c}{h} = \lim_{h\to 0} 0 = \boxed{0}$$

Thus $\boxed{f'(x) = 0 \text{ when } f(x) = c}$

Ex. 19.5 Find $\frac{df}{dx}$ where $f(x) = \frac{1}{x}$.

$$\frac{df}{dx} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$$= \lim_{h\to 0} \frac{x - x - h}{h(x)(x+h)}$$

$$= \lim_{h\to 0} \frac{-1}{x(x+h)} = \boxed{-\frac{1}{x^2}}$$

Thus $f'(x) = -\frac{1}{x^2}$ when $f(x) = \frac{1}{x}$.