Review Practice: Chapters 12 & 13

1. Find the equation of the plane through $A(0,1,-1)$, $B(1,0,-1)$, and $C(-1,1,0)$.

$\vec{AB} = \langle 1,-1,0\rangle$

$\vec{BC} = \langle -2,1,1\rangle$

$\vec{AB} \times \vec{BC} = \langle -1,-1,-\rangle$

$(x-0) + (y-1) + (z+1) = 0$

or $\boxed{x+y+z=0}$

2. Parametrize the line perpendicular to the plane with normal vector $\langle 0,1,1\rangle$ through the point $(1,2,3)$.

$x(t) = 1+0t$

$y(t) = 2+t$

$z(t) = 3+t$

or $\vec{r}(t) = \langle 1, 2+t, 3+t\rangle$

3. Find the angle between the two planes from problem 1 and 2 above. Are they orthogonal?

$\cos\theta = \dfrac{\langle 1,1,1\rangle \cdot \langle 0,1,1\rangle}{\sqrt{3}\cdot\sqrt{2}} \neq 0$ so not orthogonal

$\theta = \cos^{-1}\left(\frac{2}{\sqrt{6}}\right) \approx 35.264°$

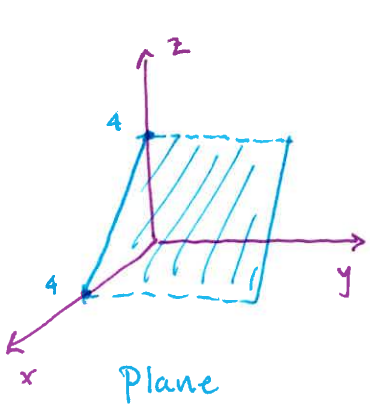
4. Sketch:

   (a) $z = 4 - x$

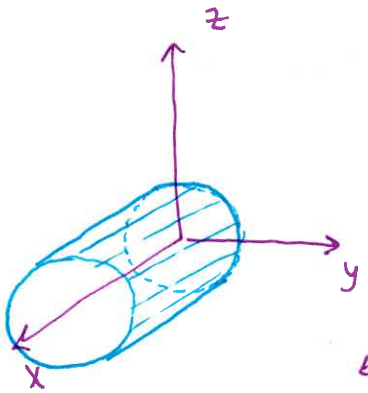
   (b) $y^2 + z^2 = 9$

   (c) $z = 4 - x^2 - y^2$

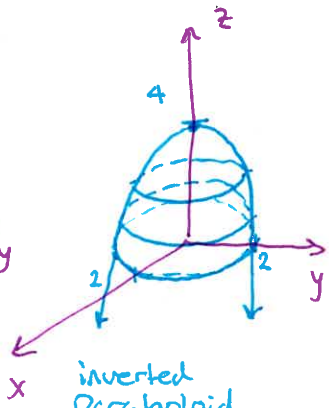
   (d) $x = z^2 + 2$



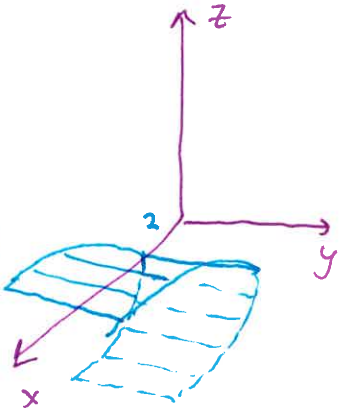
Plane



Cylinder about x-axis

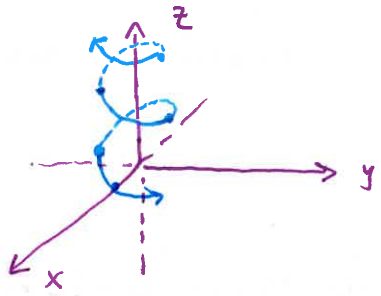


inverted Paraboloid up 4 units



Parabola about x-axis up 2 units Cylinder

5. Sketch $\mathbf{r}(t) = \langle \cos t, \sin t, -t \rangle$



Spiral up z-axis
Counter clockwise
Spiral down z-axis
Clockwise

6. Find $\lim_{t \to 0} \left\langle \frac{\sin t}{t}, t^2, \ln(t+1) \right\rangle$

$= \left\langle \lim_{t \to 0} \frac{\cos t}{1}, 0, \ln(1) \right\rangle = \boxed{\langle 1, 0, 0 \rangle}$

used L'Hopital's rule

7. Find the derivative of $\mathbf{r}(t) = \langle \sin^2 t, e^{4t} + 1, 3t^4 + t^2 \rangle$

$\boxed{\vec{r}\,'(t) = \langle 2\sin t \cdot \cos t, \; 4e^{4t}, \; 12t^3 + 2t \rangle}$

Chain Rule

8. Find the arc length of $\mathbf{r}(t) = \langle \sin(t^2), \cos(t^2), 2t^3 \rangle$ for $0 \le t \le 1$.

$$L = \int_0^1 \sqrt{(2t\cos(t^2))^2 + (-2t\sin(t^2))^2 + (6t^2)^2}\, dt$$

$$= \int_0^1 \sqrt{4t^2 + 36t^4}\, dt$$

$$= \int_0^1 2t\sqrt{1 + 9t^2}\, dt$$

$$= \frac{2}{3}\left(\frac{1+9t^2}{9}\right)^{3/2}\Bigg|_0^1$$

$$= \boxed{\frac{2}{27}\left(10^{3/2} - 1\right)}$$