Chapter 16 - Recap

MVC

- Section 16.1 - Vector fields

  Definition: function with input in $\mathbb{R}^n$ output in V^n

  Gradient Field: $\nabla f = \langle f_x, f_y, f_z \rangle$

- Section 16.2 - Line Integrals

  Line Integrals of functions: $C: \vec{r}(t) \quad a \le t \le b$

  1) WRT arc length: $\int_C f\,ds = \int_a^b f(r(t))\,|\vec{r}\,'(t)|\,dt$
  2) WRT x: $\int_C f\,dx = \int_a^b f(r(t))\left(\frac{dx}{dt}\right)dt$
  3) WRT y: $\int_C f\,dy = \int_a^b f(r(t))\left(\frac{dy}{dt}\right)dt$

  Line Integrals of vector fields:

  Notation: $\int_C \vec{F} \cdot d\vec{r}$

  Definition: $= \int_C \vec{F} \cdot \vec{T}\,ds$

  Computation: $= \int_a^b \vec{F}(r(t)) \cdot \vec{r}\,'(t)\,ds$

- Section 16.3 - FTC for Line Integrals

  FTC Part I: $\int_C \nabla f \cdot d\vec{r} = f(r(b)) - f(r(a))$

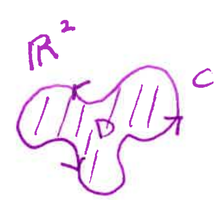
  Conservative Vector field: $\vec{F} = \nabla f$

  Potential Function: f where $\vec{F} = \nabla f$

  $\vec{F}$ conservative $\Rightarrow$ $\int_C \vec{F} \cdot d\vec{r}$ independent of path

  $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x} \Rightarrow \vec{F}$ conservative

- Section 16.4 - Green's Theorem



  Theorem: $\oint_C \vec{F} \cdot d\vec{r} = \oint_C P\,dx + Q\,dy = \iint_D Q_x - P_y\,dA$

  $A(D) = \iint_D 1\,dA = \oint_C -y\,dx = \oint_C x\,dy = \frac{1}{2}\oint_C x\,dy - y\,dx$

Chapter 16 - Recap

- Section 16.5 - Curl & Divergence $\vec{F} = \langle P, Q, R \rangle$

  Curl $\vec{F} = \nabla \times \vec{F} = \langle R_y - Q_z, P_z - R_x, Q_x - P_y \rangle$

  Curl $\nabla f = \vec{0}$

  $\vec{F}$ conservative $\Rightarrow$ Curl $\vec{F} = \vec{0}$, $\vec{F}$ defined on all $\mathbb{R}^3$ continuous partials

  Curl $\vec{F} = \vec{0} \Rightarrow \vec{F}$ conservative

  div $\vec{F} = \nabla \cdot \vec{F} = P_x + Q_y + R_z$

  div (Curl $\vec{F}$) $= 0$

- Section 16.6 - Parametric Surface & their Area

  Plane: $\vec{r_0}, \vec{a}, \vec{b}$ $\quad \vec{r}(u,v) = \vec{r_0} + \vec{a}u + \vec{b}v$

  sphere: $\vec{r}(\theta, \varphi) = \rho \langle \sin\varphi \cos\theta, \sin\varphi \sin\theta, \cos\varphi \rangle$

  Surface of Revolution: $z = f(x)$ about x axis $\vec{r}(x, \theta) = \langle x, f(x)\cos\theta, f(x)\sin\theta \rangle$

  Tangent plane: normal vector: $\vec{r_u} \times \vec{r_v}$ or $\vec{a} = \vec{r_u}$, $\vec{b} = \vec{r_v}$

  Surface Area: $A(S) = \iint_D |\vec{r_u} \times \vec{r_v}| \, dA$

  - Parametric surface: $\nearrow$
  - Graph: $A(S) = \iint_D \sqrt{f_x^2 + f_y^2 + 1} \, dA$

- Section 16.7 - Surface Integrals

  Surface Integrals of functions:

  - over parametric surface: $\iint_S f \, dS = \iint_D f(r(u,v)) |\vec{r_u} \times \vec{r_v}| \, dA$
  - Over graph: $\iint_S f \, dS = \iint_D f(g(x,y)) \sqrt{g_x^2 + g_y^2 + 1} \, dA$

  Surface Integrals of vector field:

  Notation: $\iint_S \vec{F} \cdot d\vec{S}$

  Definition: $= \iint_S \vec{F} \cdot \vec{n} \, dS$

  Computation: $= \iint_D \vec{F}(r(u,v)) \cdot (\vec{r_u} \times \vec{r_v}) \, dA$

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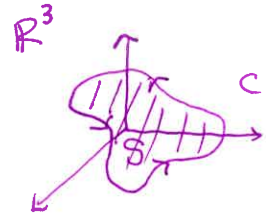
- Section 16.8 - Stoke's Theorem

Theorem: $\oint_C \vec{F}\cdot d\vec{r} = \iint_S \text{Curl}\,\vec{F}\cdot d\vec{S}$



- Section 16.9 - Divergence Theorem

Green's Theorem:

- Tangential component: $\oint_C \vec{F}\cdot d\vec{r} = \oint_C P\,dx + Q\,dy = \iint_D Q_x - P_y \, dA = \iint_D \text{Curl}\,\vec{F}\cdot d\vec{A}$
- Normal component: $\oint_C \vec{F}\cdot\vec{n}\,ds = \oint_C P\,dy - Q\,dx = \iint_D \text{Div}\,\vec{F}\,dA$

Divergence Theorem:

$$\iint_S \vec{F}\cdot d\vec{S} = \iiint_E \text{div}\,\vec{F}\,dV$$