Review Practice: Chapter 15

Formulas:

$$\sin^2\theta = \frac{1-\cos 2\theta}{2} \qquad \cos^2\theta = \frac{1+\cos 2\theta}{2}$$

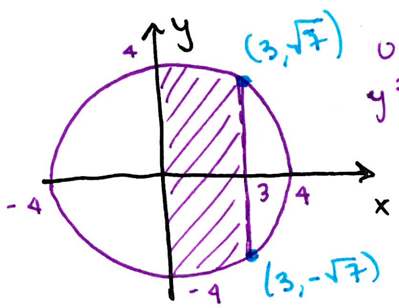
Cylindrical Coordinates:

$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$z = z$$
$$x^2 + y^2 = r^2$$
$$dA = r\ dr\ d\theta$$

Sphereical Coordinates:

$$x = \rho\sin\phi\cos\theta$$
$$y = \rho\sin\phi\sin\theta$$
$$z = \rho\cos\phi$$
$$x^2 + y^2 + z^2 = \rho^2$$
$$dV = \rho^2\sin\phi\ d\rho\ d\theta\ d\phi$$

1. Sketch the region of integration for: $\int_0^3 \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} f(x,y)\ dy\ dx$ and change the order of integration.

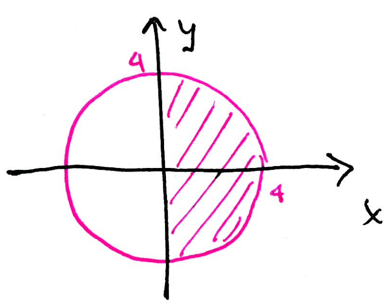


$0 \leq x \leq 3$

$y^2 \leq 16 - x^2$

$$\int_{-4}^{4}\int_{0}^{\sqrt{16-y^2}} f(x,y)\,dx\,dy - \int_{-\sqrt{7}}^{\sqrt{7}}\int_{3}^{\sqrt{16-y^2}} f(x,y)\,dx\,dy$$

2. Use polar coordinates to evaluate: $\int_0^4 \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \sec^2(x^2+y^2)\ dy\ dx$

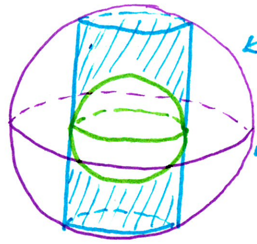


$$= \int_{-\pi/2}^{\pi/2}\int_0^4 \sec^2(r^2)\, r\,dr\,d\theta$$

$$= \pi\ \frac{\tan(r^2)}{2}\Big|_0^4$$

$$= \boxed{\frac{\pi}{2}\tan(16)}$$

3. Set up a single triple integral to find the volume between the spheres: $x^2+y^2+z^2=4$ and $x^2+y^2+z^2=1$ inside the cylinder $x^2+y^2=1$.



Two volumes that are equal

$$V = 2\int_0^{2\pi}\int_0^1\int_{\sqrt{1-r^2}}^{\sqrt{4-r^2}} 1\cdot dz\, r\,dr\,d\theta$$

4. Evaluate $\int\int\int_E z\, dV$ where E is the region described in 3.

$$= 2\int_0^{2\pi}\int_0^1\int_{\sqrt{1-r^2}}^{\sqrt{4-r^2}} z\, r\, dz\,dr\,d\theta = \frac{4\pi}{2}\int_0^1 \left[(4-r^2)-(1-r^2)\right] r\,dr$$

$$= 2\pi\left[3\right] = \boxed{6\pi}$$

5. Use spherecical coordinates to evaluate $\int\int\int_E xy\, dV$ where E is the region above the xy-plane between the spheres $x^2+y^2+z^2=4$ and $x^2+y^2+z^2=1$.

$$= \int_0^{\pi/2}\int_0^{2\pi}\int_1^4 \rho\sin\varphi\cos\theta\cdot\rho\sin\varphi\sin\theta\cdot\rho^2\sin\varphi\, d\rho\, d\theta\, d\varphi$$

$$= \int_0^{\pi/2}\sin^3\varphi\, d\varphi \cdot \overset{0}{\int_0^{2\pi}\cos\theta\,\cancel{\sin\theta\, d\theta}} \cdot \int_1^4 \rho^4\, d\rho$$

$$= \frac{1}{2}\int_0^{\pi/2}\sin\varphi - \sin\varphi\cos^2(\varphi)\, d\varphi \cdot (0)\cdot\left(\frac{4^5}{5}-\frac{1}{5}\right)$$

$$= \boxed{0}$$