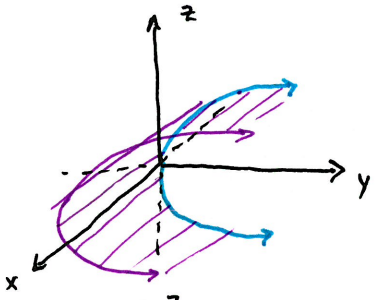
Review Practice: Chapter 12

1. Sketch the following:

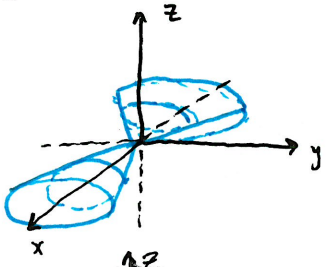
   (a) $y = z^2$

   Cylinder about x-axis



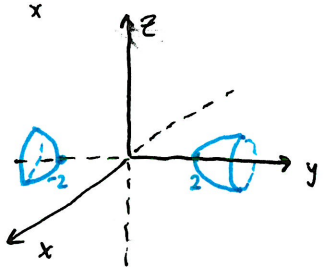
   (b) $x^2 = y^2 + 4z^2$

   Elliptic Cone about x-axis



   (c) $-4x^2 + y^2 - 4z^2 = 4$

   Hyperboloid of 2 sheets about y-axis



2. Find parametric equations for the line through $(4, -1, 2)$ and $(1, 1, 5)$.

$\vec{v} = \langle 1-4, 1+1, 5-2 \rangle = \langle -3, 2, 3 \rangle$ ← Direction

$x = 4 - 3t \qquad y = -1 + 2t \qquad z = 2 + 3t$

3. Find the equation of the plane through $(2, -1, -1)$ parallel to the plane $x + 4y - 3z = 1$.

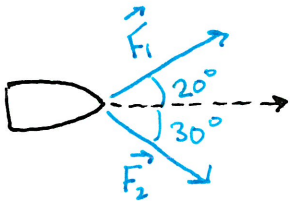
$\vec{n} = \langle 1, 4, -3 \rangle$

$1(x-2) + 4(y+1) - 3(z+1) = 0$

or

$x + 4y - 3z = 1$

4. A boart is pulled onto shore using 2 ropes, one at an angle of 20° and the other at an angle of 30° from the front center of the boat. If a force of 255N is needed, find the magnitude of the force in each rope.



Horizontal Forces:

① $F_1 \cos 20° + F_2 \cos 30° = 255$

Vertical Forces:

② $F_1 \sin 20° = F_2 \sin 30°$

② : $F_1 = F_2 \frac{\sin 30°}{\sin 20°}$

② into ① : $F_2 (\sin 30° \cot 20° + \cos 30°) = 255$

$F_2 \approx 113.85\text{N}$

$F_1 = 166.44\text{N}$

5. State whether the result is a vector or scalar if defined, otherwise state not defined:

(a) $(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d})$ → Yes, Scalar since $\vec{a}\times\vec{b}$, $\vec{c}\times\vec{d}$ are vectors and dot product is defined for two vectors.

(b) $(\mathbf{a} \cdot \mathbf{b}) \times (\mathbf{c} \cdot \mathbf{d})$ → Not defined since $\vec{a}\cdot\vec{b}$, $\vec{c}\cdot\vec{d}$ are scalars and cross product is not defined for two scalars.

(c) $(\mathbf{a} \times \mathbf{b}) \times (\mathbf{c} \times \mathbf{d})$ → Yes, Vector since $\vec{a}\times\vec{b}$, $\vec{c}\times\vec{d}$ are vectors and cross product of two vectors returns a vector.

6. Find x so that $\langle 3x, 0, 1+x \rangle$ and $\langle 1+x, 1-x, 1 \rangle$ are orthogonal. Is there any x so that they are parallel?

Orthogonal: $0 = \langle 3x, 0, 1+x \rangle \cdot \langle 1+x, 1-x, 1 \rangle$

$= 3x + 3x^2 + 1 + x$

$= 3x^2 + 4x + 1$

$= (3x+1)(x+1)$ $\quad x = -\frac{1}{3}, -1$

Parallel: $\vec{0} = \langle 3x, 0, 1+x \rangle \times \langle 1+x, 1-x, 1 \rangle$

$= \langle -(1+x)(1-x),\ (1+x)^2 - 3x,\ 3x(1-x) \rangle$

$x = \pm 1 \longrightarrow x = 1 \longleftarrow x = 0, 1$

but $(1+1)^2 - 3(1) \neq 0$

Thus there is no x for which they will be parallel.