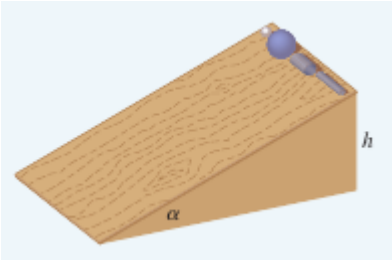
Roller Derby Project

We are going to race 4 objects down a ramp. Suppose you roll a marble (solid ball), a squash ball (hollow ball), streel bar (solid cylinder), and a lead pipe (hollow cylinder) down a ramp at the same time.



1. Make a guess about which object will reach the bottom first.
2. To answer definitely we consider a ball and cylinder with mass m, radius r, moment of inertia I (about the axis of rotation). If the highest place on the ramp is h, then what is the potential energy of the object?
3. If the object reaches the bottom with velocity v and angular velocity ω (with $v = r\omega$) then what is the kinetic energy at the bottom of the ramp? (Hint: Remember that kinentic energy consists of translational and rotational energy.)
4. If the energy lost by friction is negligible then conservation of energy gives:
5. Show that $v^2 = \dfrac{2gh}{1+I^*}$ where $I^* = \dfrac{I}{mr^2}$.
6. Let $y(t)$ be the vertical distance traveled at time t. Show that $v^2 = \dfrac{2gy(t)}{1+I^*}$ at any time t.
7. Show that y satisfies the differential equation:

$$\frac{dy}{dt} = \sqrt{\frac{2g}{1+I^*}}(\sin\alpha)\sqrt{y}$$

   where α is the angle of inclination of the ramp.
8. Solve the differential equation above and use it to find the total travel time T.
9. Since h, g, α are constants what must be true about I^* to minimize the total travel time T and hence win the race?
10. Find I^* for a solid cylinder.
11. Find I^* for a hollow cylinder.
12. Find I^* for a solid sphere.
13. Find I^* for a hollow sphere.
14. Conclude the finishing order of the race.