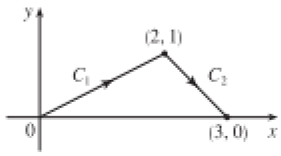
Section 16.2 - 3, 7, 11, 15, 17, 20, 33, 41

3. Parametric equations for C are $x = 4\cos t$, $y = 4\sin t$, $-\frac{\pi}{2} \le t \le \frac{\pi}{2}$. Then

$$\int_C xy^4\,ds = \int_{-\pi/2}^{\pi/2}(4\cos t)(4\sin t)^4\sqrt{(-4\sin t)^2 + (4\cos t)^2}\,dt = \int_{-\pi/2}^{\pi/2} 4^5\cos t\,\sin^4 t\,\sqrt{16(\sin^2 t + \cos^2 t)}\,dt$$
$$= 4^5\int_{-\pi/2}^{\pi/2}(\sin^4 t\cos t)(4)\,dt = (4)^6\left[\tfrac{1}{5}\sin^5 t\right]_{-\pi/2}^{\pi/2} = \frac{2\cdot 4^6}{5} = 1638.4$$

7.



$C = C_1 + C_2$

On C_1: $x = x, y = \frac{1}{2}x \;\Rightarrow\; dy = \frac{1}{2}\,dx$, $0 \le x \le 2$.

On C_2: $x = x, y = 3 - x \;\Rightarrow\; dy = -dx$, $2 \le x \le 3$.

Then

$$\int_C (x+2y)\,dx + x^2\,dy = \int_{C_1}(x+2y)\,dx + x^2\,dy + \int_{C_2}(x+2y)\,dx + x^2\,dy$$
$$= \int_0^2 \left[x + 2\left(\tfrac{1}{2}x\right) + x^2\left(\tfrac{1}{2}\right)\right]dx + \int_2^3 \left[x + 2(3-x) + x^2(-1)\right]dx$$
$$= \int_0^2 \left(2x + \tfrac{1}{2}x^2\right)dx + \int_2^3 \left(6 - x - x^2\right)dx$$
$$= \left[x^2 + \tfrac{1}{6}x^3\right]_0^2 + \left[6x - \tfrac{1}{2}x^2 - \tfrac{1}{3}x^3\right]_2^3 = \tfrac{16}{3} - 0 + \tfrac{9}{2} - \tfrac{22}{3} = \tfrac{5}{2}$$

11. Parametric equations for C are $x = t$, $y = 2t$, $z = 3t$, $0 \le t \le 1$. Then

$$\int_C xe^{yz}\,ds = \int_0^1 te^{(2t)(3t)}\sqrt{1^2+2^2+3^2}\,dt = \sqrt{14}\int_0^1 te^{6t^2}\,dt = \sqrt{14}\left[\tfrac{1}{12}e^{6t^2}\right]_0^1 = \tfrac{\sqrt{14}}{12}(e^6 - 1).$$

15. Parametric equations for C are $x = 1 + 3t$, $y = t$, $z = 2t$, $0 \le t \le 1$. Then

$$\int_C z^2\,dx + x^2\,dy + y^2\,dz = \int_0^1 (2t)^2\cdot 3\,dt + (1+3t)^2\,dt + t^2\cdot 2\,dt = \int_0^1 \left(23t^2 + 6t + 1\right)dt$$
$$= \left[\tfrac{23}{3}t^3 + 3t^2 + t\right]_0^1 = \tfrac{23}{3} + 3 + 1 = \tfrac{35}{3}$$

17. (a) Along the line $x = -3$, the vectors of $\mathbf{F}$ have positive y-components, so since the path goes upward, the integrand $\mathbf{F}\cdot\mathbf{T}$ is always positive. Therefore $\int_{C_1}\mathbf{F}\cdot d\mathbf{r} = \int_{C_1}\mathbf{F}\cdot\mathbf{T}\,ds$ is positive.

(b) All of the (nonzero) field vectors along the circle with radius 3 are pointed in the clockwise direction, that is, opposite the direction to the path. So $\mathbf{F}\cdot\mathbf{T}$ is negative, and therefore $\int_{C_2}\mathbf{F}\cdot d\mathbf{r} = \int_{C_2}\mathbf{F}\cdot\mathbf{T}\,ds$ is negative.

20. $\mathbf{F}(\mathbf{r}(t)) = (t^2+t^3)\,\mathbf{i} + (t^3-t^2)\,\mathbf{j} + (t^2)^2\mathbf{k} = (t^2+t^3)\,\mathbf{i} + (t^3-t^2)\,\mathbf{j} + t^4\,\mathbf{k}$, $\mathbf{r}'(t) = 2t\,\mathbf{i} + 3t^2\,\mathbf{j} + 2t\,\mathbf{k}$. Then

$$\int_C \mathbf{F}\cdot d\mathbf{r} = \int_0^1 \mathbf{F}(\mathbf{r}(t))\cdot\mathbf{r}'(t)\,dt = \int_0^1 (2t^3 + 2t^4 + 3t^5 - 3t^4 + 2t^5)\,dt = \int_0^1 (5t^5 - t^4 + 2t^3)\,dt$$
$$= \left[\tfrac{5}{6}t^6 - \tfrac{1}{5}t^5 + \tfrac{1}{2}t^4\right]_0^1 = \tfrac{5}{6} - \tfrac{1}{5} + \tfrac{1}{2} = \tfrac{17}{15}.$$

33. We use the parametrization $x = 2\cos t$, $y = 2\sin t$, $-\frac{\pi}{2} \le t \le \frac{\pi}{2}$. Then

$ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt = \sqrt{(-2\sin t)^2 + (2\cos t)^2}\,dt = 2\,dt$, so $m = \int_C k\,ds = 2k\int_{-\pi/2}^{\pi/2} dt = 2k(\pi)$,

$\overline{x} = \frac{1}{2\pi k}\int_C xk\,ds = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2}(2\cos t)2\,dt = \frac{1}{2\pi}\left[4\sin t\right]_{-\pi/2}^{\pi/2} = \frac{4}{\pi}$, $\overline{y} = \frac{1}{2\pi k}\int_C yk\,ds = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2}(2\sin t)2\,dt = 0$.

Hence $(\overline{x}, \overline{y}) = \left(\frac{4}{\pi}, 0\right)$.

41. $\mathbf{r}(t) = \langle 2t, t, 1-t \rangle$, $0 \le t \le 1$.

$$\begin{aligned} W &= \int_C \mathbf{F}\cdot d\mathbf{r} = \int_0^1 \left\langle 2t - t^2, t - (1-t)^2, 1 - t - (2t)^2 \right\rangle \cdot \langle 2, 1, -1 \rangle\,dt \\ &= \int_0^1 \left(4t - 2t^2 + t - 1 + 2t - t^2 - 1 + t + 4t^2\right)dt = \int_0^1 \left(t^2 + 8t - 2\right)dt = \left[\tfrac{1}{3}t^3 + 4t^2 - 2t\right]_0^1 = \tfrac{7}{3} \end{aligned}$$