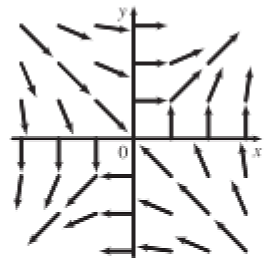
Section 16.1 - 5, 9, 11-14, 15-18, 21, 25

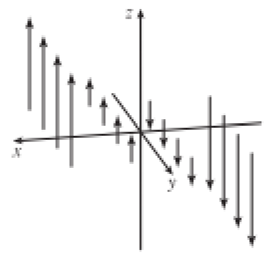
5. $\mathbf{F}(x,y) = \dfrac{y\,\mathbf{i} + x\,\mathbf{j}}{\sqrt{x^2+y^2}}$

The length of the vector $\dfrac{y\,\mathbf{i} + x\,\mathbf{j}}{\sqrt{x^2+y^2}}$ is 1.



9. $\mathbf{F}(x,y,z) = x\,\mathbf{k}$

At each point (x,y,z), $\mathbf{F}(x,y,z)$ is a vector of length $|x|$. For $x > 0$, all point in the direction of the positive z-axis, while for $x < 0$, all are in the direction of the negative z-axis. In each plane $x = k$, all the vectors are identical.



11. $\mathbf{F}(x,y) = \langle x, -y \rangle$ corresponds to graph IV. In the first quadrant all the vectors have positive x-components and negative y-components, in the second quadrant all vectors have negative x- and y-components, in the third quadrant all vectors have negative x-components and positive y-components, and in the fourth quadrant all vectors have positive x- and y-components. In addition, the vectors get shorter as we approach the origin.

12. $\mathbf{F}(x,y) = \langle y, x-y \rangle$ corresponds to graph III. All vectors in quadrants I and II have positive x-components while all vectors in quadrants III and IV have negative x-components. In addition, vectors along the line $y = x$ are horizontal, and vectors get shorter as we approach the origin.

13. $\mathbf{F}(x,y) = \langle y, y+2 \rangle$ corresponds to graph I. As in Exercise 12, all vectors in quadrants I and II have positive x-components while all vectors in quadrants III and IV have negative x-components.Vectors along the line $y = -2$ are horizontal, and the vectors are independent of x (vectors along horizontal lines are identical).

14. $\mathbf{F}(x,y) = \langle \cos(x+y), x \rangle$ corresponds to graph II. All vectors in quadrants I and IV have positive y-components while all vectors in quadrants II and III have negative y-components. Also, the y-components of vectors along any vertical line remain constant while the x-component oscillates.

15. $\mathbf{F}(x, y, z) = \mathbf{i} + 2\,\mathbf{j} + 3\,\mathbf{k}$ corresponds to graph IV, since all vectors have identical length and direction.

16. $\mathbf{F}(x, y, z) = \mathbf{i} + 2\,\mathbf{j} + z\,\mathbf{k}$ corresponds to graph I, since the horizontal vector components remain constant, but the vectors above the xy-plane point generally upward while the vectors below the xy-plane point generally downward.

17. $\mathbf{F}(x, y, z) = x\,\mathbf{i} + y\,\mathbf{j} + 3\,\mathbf{k}$ corresponds to graph III; the projection of each vector onto the xy-plane is $x\,\mathbf{i} + y\,\mathbf{j}$, which points away from the origin, and the vectors point generally upward because their z-components are all 3.

18. $\mathbf{F}(x, y, z) = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$ corresponds to graph II; each vector $\mathbf{F}(x, y, z)$ has the same length and direction as the position vector of the point (x, y, z), and therefore the vectors all point directly away from the origin.

21. $f(x, y) = xe^{xy} \quad \Rightarrow$

$$\nabla f(x, y) = f_x(x, y)\,\mathbf{i} + f_y(x, y)\,\mathbf{j} = (xe^{xy} \cdot y + e^{xy})\,\mathbf{i} + (xe^{xy} \cdot x)\,\mathbf{j} = (xy + 1)e^{xy}\,\mathbf{i} + x^2 e^{xy}\,\mathbf{j}$$

25. $f(x, y) = x^2 - y \quad \Rightarrow \quad \nabla f(x, y) = 2x\,\mathbf{i} - \mathbf{j}$.

The length of $\nabla f(x, y)$ is $\sqrt{4x^2 + 1}$. When $x \neq 0$, the vectors point away from the y-axis in a slightly downward direction with length that increases as the distance from the y-axis increases.