Section 15.2 - 9, 12, 17, 22, 25, 27, 36

9. $\int_1^4 \int_1^2 \left(\frac{x}{y}+\frac{y}{x}\right) dy\,dx = \int_1^4 \left[x\ln|y| + \frac{1}{x}\cdot\frac{1}{2}y^2\right]_{y=1}^{y=2} dx = \int_1^4 \left(x\ln 2 + \frac{3}{2x}\right) dx = \left[\frac{1}{2}x^2\ln 2 + \frac{3}{2}\ln|x|\right]_1^4$

$= 8\ln 2 + \frac{3}{2}\ln 4 - \frac{1}{2}\ln 2 = \frac{15}{2}\ln 2 + 3\ln 4^{1/2} = \frac{21}{2}\ln 2$

12. $\int_0^1\int_0^1 xy\sqrt{x^2+y^2}\,dy\,dx = \int_0^1 x\left[\frac{1}{3}(x^2+y^2)^{3/2}\right]_{y=0}^{y=1} dx = \frac{1}{3}\int_0^1 x[(x^2+1)^{3/2} - x^3]\,dx = \frac{1}{3}\int_0^1 [x(x^2+1)^{3/2} - x^4]dx$

$= \frac{1}{3}\left[\frac{1}{5}(x^2+1)^{5/2} - \frac{1}{5}x^5\right]_0^1 = \frac{1}{15}\left[2^{5/2} - 1 - 1 + 0\right] = \frac{2}{15}(2\sqrt{2}-1)$

17. $\iint_R \frac{xy^2}{x^2+1}\,dA = \int_0^1 \int_{-3}^3 \frac{xy^2}{x^2+1}\,dy\,dx = \int_0^1 \frac{x}{x^2+1}\,dx \int_{-3}^3 y^2\,dy = \left[\frac{1}{2}\ln(x^2+1)\right]_0^1 \left[\frac{1}{3}y^3\right]_{-3}^3$

$= \frac{1}{2}(\ln 2 - \ln 1)\cdot\frac{1}{3}(27+27) = 9\ln 2$

22. $\iint_R \frac{1}{1+x+y}\,dA = \int_1^3 \int_1^2 \frac{1}{1+x+y}\,dy\,dx = \int_1^3 [\ln(1+x+y)]_{y=1}^{y=2}\,dx = \int_1^3 [\ln(x+3) - \ln(x+2)]\,dx$

$= \left[\left((x+3)\ln(x+3) - (x+3)\right) - \left((x+2)\ln(x+2) - (x+2)\right)\right]_1^3$

[by integrating by parts separately for each term]

$= (6\ln 6 - 6 - 5\ln 5 + 5) - (4\ln 4 - 4 - 3\ln 3 + 3) = 6\ln 6 - 5\ln 5 - 4\ln 4 + 3\ln 3$

25. The solid lies under the plane $4x + 6y - 2z + 15 = 0$ or $z = 2x + 3y + \frac{15}{2}$ so

$V = \iint_R (2x + 3y + \frac{15}{2})\,dA = \int_{-1}^1 \int_{-1}^2 (2x + 3y + \frac{15}{2})\,dx\,dy = \int_{-1}^1 \left[x^2 + 3xy + \frac{15}{2}x\right]_{x=-1}^{x=2}\,dy$

$= \int_{-1}^1 \left[(19+6y) - (-\frac{13}{2} - 3y)\right]dy = \int_{-1}^1 (\frac{51}{2} + 9y)\,dy = \left[\frac{51}{2}y + \frac{9}{2}y^2\right]_{-1}^1 = 30 - (-21) = 51$

27. $V = \int_{-2}^2 \int_{-1}^1 \left(1 - \frac{1}{4}x^2 - \frac{1}{9}y^2\right) dx\,dy = 4\int_0^2\int_0^1 \left(1 - \frac{1}{4}x^2 - \frac{1}{9}y^2\right) dx\,dy$

$= 4\int_0^2 \left[x - \frac{1}{12}x^3 - \frac{1}{9}y^2 x\right]_{x=0}^{x=1} dy = 4\int_0^2 \left(\frac{11}{12} - \frac{1}{9}y^2\right) dy = 4\left[\frac{11}{12}y - \frac{1}{27}y^3\right]_0^2 = 4\cdot\frac{83}{54} = \frac{166}{27}$

36. $A(R) = 4\cdot 1 = 4$, so

$f_{ave} = \frac{1}{A(R)}\iint_R f(x,y)\,dA = \frac{1}{4}\int_0^4 \int_0^1 e^y\sqrt{x+e^y}\,dy\,dx = \frac{1}{4}\int_0^4 \left[\frac{2}{3}(x+e^y)^{3/2}\right]_{y=0}^{y=1} dx$

$= \frac{1}{4}\cdot\frac{2}{3}\int_0^4 [(x+e)^{3/2} - (x+1)^{3/2}]\,dx = \frac{1}{6}\left[\frac{2}{5}(x+e)^{5/2} - \frac{2}{5}(x+1)^{5/2}\right]_0^4$

$= \frac{1}{6}\cdot\frac{2}{5}[(4+e)^{5/2} - 5^{5/2} - e^{5/2} + 1] = \frac{1}{15}[(4+e)^{5/2} - e^{5/2} - 5^{5/2} + 1] \approx 3.327$