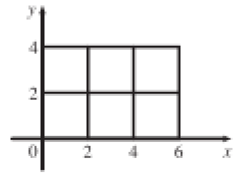
Section 15.1 - 1, 4, 5, 13, 18

1. (a) The subrectangles are shown in the figure.



The surface is the graph of $f(x,y) = xy$ and $\Delta A = 4$, so we estimate

$$V \approx \sum_{i=1}^{3} \sum_{j=1}^{2} f(x_i, y_j)\,\Delta A$$

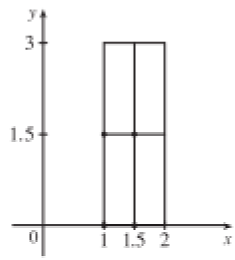
$$= f(2,2)\,\Delta A + f(2,4)\,\Delta A + f(4,2)\,\Delta A + f(4,4)\,\Delta A + f(6,2)\,\Delta A + f(6,4)\,\Delta A$$

$$= 4(4) + 8(4) + 8(4) + 16(4) + 12(4) + 24(4) = 288$$

(b) $V \approx \sum_{i=1}^{3} \sum_{j=1}^{2} f(\overline{x}_i, \overline{y}_j)\,\Delta A = f(1,1)\,\Delta A + f(1,3)\,\Delta A + f(3,1)\,\Delta A + f(3,3)\,\Delta A + f(5,1)\,\Delta A + f(5,3)\,\Delta A$

$$= 1(4) + 3(4) + 3(4) + 9(4) + 5(4) + 15(4) = 144$$

4. (a) The subrectangles are shown in the figure.



The surface is the graph of $f(x,y) = 1 + x^2 + 3y$ and $\Delta A = \frac{1}{2} \cdot \frac{3}{2} = \frac{3}{4}$,

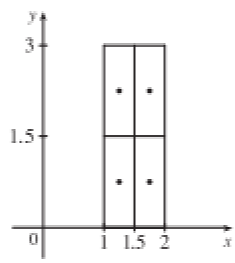
so we estimate

$$V = \iint_R (1 + x^2 + 3y)\,dA \approx \sum_{i=1}^{2} \sum_{j=1}^{2} f(x_{ij}^*, y_{ij}^*)\,\Delta A$$

$$= f(1,0)\,\Delta A + f\left(1, \tfrac{3}{2}\right)\Delta A + f\left(\tfrac{3}{2}, 0\right)\Delta A + f\left(\tfrac{3}{2}, \tfrac{3}{2}\right)\Delta A$$

$$= 2\left(\tfrac{3}{4}\right) + \tfrac{13}{2}\left(\tfrac{3}{4}\right) + \tfrac{13}{4}\left(\tfrac{3}{4}\right) + \tfrac{31}{4}\left(\tfrac{3}{4}\right) = \tfrac{39}{2}\left(\tfrac{3}{4}\right) = \tfrac{117}{8} = 14.625$$

(b) $V = \iint_R (1 + x^2 + 3y)\,dA \approx \sum_{i=1}^{2} \sum_{j=1}^{2} f(\overline{x}_i, \overline{y}_j)\,\Delta A$

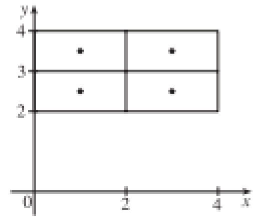


$$= f\left(\tfrac{5}{4}, \tfrac{3}{4}\right)\Delta A + f\left(\tfrac{5}{4}, \tfrac{9}{4}\right)\Delta A + f\left(\tfrac{7}{4}, \tfrac{3}{4}\right)\Delta A + f\left(\tfrac{7}{4}, \tfrac{9}{4}\right)\Delta A$$

$$= \tfrac{77}{16}\left(\tfrac{3}{4}\right) + \tfrac{149}{16}\left(\tfrac{3}{4}\right) + \tfrac{101}{16}\left(\tfrac{3}{4}\right) + \tfrac{173}{16}\left(\tfrac{3}{4}\right) = \tfrac{375}{16} = 23.4375$$

5. (a) Each subrectangle and its midpoint are shown in the figure.

The area of each subrectangle is $\Delta A = 2$, so we evaluate f at each midpoint and estimate



$$\iint_R f(x,y)\,dA \approx \sum_{i=1}^{2}\sum_{j=1}^{2} f(\overline{x}_i,\overline{y}_j)\,\Delta A$$

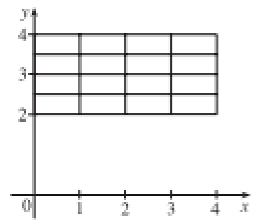
$$= f(1,2.5)\,\Delta A + f(1,3.5)\,\Delta A + f(3,2.5)\,\Delta A + f(3,3.5)\,\Delta A$$

$$= -2(2) + (-1)(2) + 2(2) + 3(2) = 4$$

(b) The subrectangles are shown in the figure.

In each subrectangle, the sample point closest to the origin is the lower left corner, and the area of each subrectangle is $\Delta A = \frac{1}{2}$.

Thus we estimate



$$\iint_R f(x,y)\,dA \approx \sum_{i=1}^{4}\sum_{j=1}^{4} f(x_{ij}^*,y_{ij}^*)\,\Delta A$$

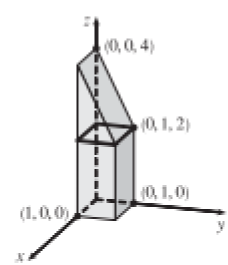
$$= f(0,2)\,\Delta A + f(0,2.5)\,\Delta A + f(0,3)\,\Delta A + f(0,3.5)\,\Delta A$$
$$+ f(1,2)\,\Delta A + f(1,2.5)\,\Delta A + f(1,3)\,\Delta A + f(1,3.5)\,\Delta A$$
$$+ f(2,2)\,\Delta A + f(2,2.5)\,\Delta A + f(2,3)\,\Delta A + f(2,3.5)\,\Delta A$$
$$+ f(3,2)\,\Delta A + f(3,2.5)\,\Delta A + f(3,3)\,\Delta A + f(3,3.5)\,\Delta A$$

$$= -3(\tfrac{1}{2}) + (-5)(\tfrac{1}{2}) + (-6)(\tfrac{1}{2}) + (-4)(\tfrac{1}{2}) + (-1)(\tfrac{1}{2}) + (-2)(\tfrac{1}{2}) + (-3)(\tfrac{1}{2}) + (-1)(\tfrac{1}{2})$$
$$+ 1(\tfrac{1}{2}) + 0(\tfrac{1}{2}) + (-1)(\tfrac{1}{2}) + 1(\tfrac{1}{2}) + 2(\tfrac{1}{2}) + 2(\tfrac{1}{2}) + 1(\tfrac{1}{2}) + 3(\tfrac{1}{2})$$

$$= -8$$

13. $z = f(x,y) = 4 - 2y \geq 0$ for $0 \leq y \leq 1$. Thus the integral represents the volume of that part of the rectangular solid $[0,1] \times [0,1] \times [0,4]$ which lies below the plane $z = 4 - 2y$. So

$$\iint_R (4-2y)\,dA = (1)(1)(2) + \tfrac{1}{2}(1)(1)(2) = 3$$



18. Because $\sin \pi x$ is an increasing function for $0 \leq x \leq \frac{1}{4}$, we have $\sin 0 \leq \sin \pi x \leq \sin \frac{\pi}{4} \quad \Rightarrow \quad 0 \leq \sin \pi x \leq \frac{\sqrt{2}}{2}$. Similarly, $\cos \pi y$ is a decreasing function for $\frac{1}{4} \leq y \leq \frac{1}{2}$, so $0 = \cos \frac{\pi}{2} \leq \cos \pi y \leq \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$. Thus on R, $0 \leq \sin \pi x \, \cos \pi y \leq \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{1}{2}$. Property (9) gives $\iint_R 0\,dA \leq \iint_R \sin \pi x \cos \pi y\,dA \leq \iint_R \frac{1}{2}\,dA$, so by Exercise 17 we have $0 \leq \iint_R \sin \pi x \, \cos \pi y\,dA \leq \frac{1}{2}\left(\frac{1}{4} - 0\right)\left(\frac{1}{2} - \frac{1}{4}\right) = \frac{1}{32}$.