Section 14.4 - 4, 15, 19, 21, 28, 33, 35, 39

4. $z = f(x,y) = xe^{xy} \quad \Rightarrow \quad f_x(x,y) = xye^{xy} + e^{xy},\ f_y(x,y) = x^2e^{xy},$ so $f_x(2,0) = 1,\ f_y(2,0) = 4$, and an equation of the tangent plane is $z - 2 = f_x(2,0)(x-2) + f_y(2,0)(y-0) \quad \Rightarrow \quad z - 2 = 1(x-2) + 4(y-0)$ or $z = x + 4y$.

15. $f(x,y) = e^{-xy}\cos y$. The partial derivatives are $f_x(x,y) = e^{-xy}(-y)\cos y = -ye^{-xy}\cos y$ and $f_y(x,y) = e^{-xy}(-\sin y) + (\cos y)e^{-xy}(-x) = -e^{-xy}(\sin y + x\cos y)$, so $f_x(\pi,0) = 0$ and $f_y(\pi,0) = -\pi$. Both f_x and f_y are continuous functions, so f is differentiable at $(\pi,0)$, and the linearization of f at $(\pi,0)$ is $L(x,y) = f(\pi,0) + f_x(\pi,0)(x-\pi) + f_y(\pi,0)(y-0) = 1 + 0(x-\pi) - \pi(y-0) = 1 - \pi y$.

19. We can estimate $f(2.2, 4.9)$ using a linear approximation of f at $(2,5)$, given by $f(x,y) \approx f(2,5) + f_x(2,5)(x-2) + f_y(2,5)(y-5) = 6 + 1(x-2) + (-1)(y-5) = x - y + 9$. Thus $f(2.2, 4.9) \approx 2.2 - 4.9 + 9 = 6.3$.

21. $f(x,y,z) = \sqrt{x^2+y^2+z^2} \quad \Rightarrow \quad f_x(x,y,z) = \dfrac{x}{\sqrt{x^2+y^2+z^2}},\ f_y(x,y,z) = \dfrac{y}{\sqrt{x^2+y^2+z^2}}$, and $f_z(x,y,z) = \dfrac{z}{\sqrt{x^2+y^2+z^2}}$, so $f_x(3,2,6) = \frac{3}{7},\ f_y(3,2,6) = \frac{2}{7},\ f_z(3,2,6) = \frac{6}{7}$. Then the linear approximation of f at $(3,2,6)$ is given by

$$\begin{aligned} f(x,y,z) &\approx f(3,2,6) + f_x(3,2,6)(x-3) + f_y(3,2,6)(y-2) + f_z(3,2,6)(z-6) \\ &= 7 + \tfrac{3}{7}(x-3) + \tfrac{2}{7}(y-2) + \tfrac{6}{7}(z-6) = \tfrac{3}{7}x + \tfrac{2}{7}y + \tfrac{6}{7}z \end{aligned}$$

Thus $\sqrt{(3.02)^2 + (1.97)^2 + (5.99)^2} = f(3.02, 1.97, 5.99) \approx \frac{3}{7}(3.02) + \frac{2}{7}(1.97) + \frac{6}{7}(5.99) \approx 6.9914$.

28. $T = \dfrac{v}{1+uvw} \quad \Rightarrow$

$$\begin{aligned} dT &= \frac{\partial T}{\partial u}\,du + \frac{\partial T}{\partial v}\,dv + \frac{\partial T}{\partial w}\,dw \\ &= v(-1)(1+uvw)^{-2}(vw)\,du + \frac{1(1+uvw) - v(uw)}{(1+uvw)^2}\,dv + v(-1)(1+uvw)^{-2}(uv)\,dw \\ &= -\frac{v^2w}{(1+uvw)^2}\,du + \frac{1}{(1+uvw)^2}\,dv - \frac{uv^2}{(1+uvw)^2}\,dw \end{aligned}$$

33. $dA = \dfrac{\partial A}{\partial x}\,dx + \dfrac{\partial A}{\partial y}\,dy = y\,dx + x\,dy$ and $|\Delta x| \le 0.1$, $|\Delta y| \le 0.1$. We use $dx = 0.1$, $dy = 0.1$ with $x = 30$, $y = 24$; then the maximum error in the area is about $dA = 24(0.1) + 30(0.1) = 5.4\ \text{cm}^2$.

35. The volume of a can is $V = \pi r^2 h$ and $\Delta V \approx dV$ is an estimate of the amount of tin. Here $dV = 2\pi rh\,dr + \pi r^2\,dh$, so put $dr = 0.04$, $dh = 0.08$ (0.04 on top, 0.04 on bottom) and then $\Delta V \approx dV = 2\pi(48)(0.04) + \pi(16)(0.08) \approx 16.08\ \text{cm}^3$. Thus the amount of tin is about $16\ \text{cm}^3$.

39. First we find $\dfrac{\partial R}{\partial R_1}$ implicitly by taking partial derivatives of both sides with respect to R_1:

$\dfrac{\partial}{\partial R_1}\left(\dfrac{1}{R}\right) = \dfrac{\partial\left[(1/R_1)+(1/R_2)+(1/R_3)\right]}{\partial R_1} \quad\Rightarrow\quad -R^{-2}\dfrac{\partial R}{\partial R_1} = -R_1^{-2} \quad\Rightarrow\quad \dfrac{\partial R}{\partial R_1} = \dfrac{R^2}{R_1^2}$. Then by symmetry,

$\dfrac{\partial R}{\partial R_2} = \dfrac{R^2}{R_2^2}$, $\dfrac{\partial R}{\partial R_3} = \dfrac{R^2}{R_3^2}$. When $R_1 = 25$, $R_2 = 40$ and $R_3 = 50$, $\dfrac{1}{R} = \dfrac{17}{200} \quad\Leftrightarrow\quad R = \frac{200}{17}\,\Omega$. Since the possible error for each R_i is 0.5%, the maximum error of R is attained by setting $\Delta R_i = 0.005R_i$. So

$$\Delta R \approx dR = \frac{\partial R}{\partial R_1}\Delta R_1 + \frac{\partial R}{\partial R_2}\Delta R_2 + \frac{\partial R}{\partial R_3}\Delta R_3 = (0.005)R^2\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right) = (0.005)R = \tfrac{1}{17} \approx 0.059\,\Omega.$$