Section 14.2 - 7, 11, 16, 17, 26, 29, 34, 37

7. $f(x,y) = \dfrac{4-xy}{x^2+3y^2}$ is a rational function and hence continuous on its domain.

$(2,1)$ is in the domain of f, so f is continuous there and $\lim\limits_{(x,y)\to(2,1)} f(x,y) = f(2,1) = \dfrac{4-(2)(1)}{(2)^2+3(1)^2} = \dfrac{2}{7}$.

11. $f(x,y) = (y^2\sin^2 x)/(x^4+y^4)$. On the x-axis, $f(x,0)=0$ for $x \neq 0$, so $f(x,y) \to 0$ as $(x,y) \to (0,0)$ along the x-axis. Approaching $(0,0)$ along the line $y = x$, $f(x,x) = \dfrac{x^2\sin^2 x}{x^4+x^4} = \dfrac{\sin^2 x}{2x^2} = \dfrac{1}{2}\left(\dfrac{\sin x}{x}\right)^2$ for $x \neq 0$ and $\lim\limits_{x\to 0} \dfrac{\sin x}{x} = 1$, so $f(x,y) \to \frac{1}{2}$. Since f has two different limits along two different lines, the limit does not exist.

16. We can use the Squeeze Theorem to show that $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2\sin^2 y}{x^2+2y^2} = 0$:

$0 \le \dfrac{x^2\sin^2 y}{x^2+2y^2} \le \sin^2 y$ since $\dfrac{x^2}{x^2+2y^2} \le 1$, and $\sin^2 y \to 0$ as $(x,y)\to(0,0)$, so $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2\sin^2 y}{x^2+2y^2} = 0$.

17.
$$\begin{aligned}\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{\sqrt{x^2+y^2+1}-1} &= \lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{\sqrt{x^2+y^2+1}-1}\cdot\frac{\sqrt{x^2+y^2+1}+1}{\sqrt{x^2+y^2+1}+1}\\ &= \lim_{(x,y)\to(0,0)} \frac{(x^2+y^2)\left(\sqrt{x^2+y^2+1}+1\right)}{x^2+y^2} = \lim_{(x,y)\to(0,0)} \left(\sqrt{x^2+y^2+1}+1\right) = 2\end{aligned}$$

26. $h(x,y) = g(f(x,y)) = \dfrac{1-xy}{1+x^2y^2} + \ln\left(\dfrac{1-xy}{1+x^2y^2}\right)$. f is a rational function, so it is continuous on its domain. Because $1+x^2y^2 > 0$, the domain of f is $\mathbb{R}^2$, so f is continuous everywhere. g is continuous on its domain $\{t \mid t > 0\}$. Thus h is continuous on its domain $\left\{(x,y) \;\middle|\; \dfrac{1-xy}{1+x^2y^2} > 0\right\} = \{(x,y) \mid xy < 1\}$ which consists of all points between (but not on) the two branches of the hyperbola $y = 1/x$.

29. The functions xy and $1+e^{x-y}$ are continuous everywhere, and $1+e^{x-y}$ is never zero, so $F(x,y) = \dfrac{xy}{1+e^{x-y}}$ is continuous on its domain $\mathbb{R}^2$.

34. $G(x,y) = g(f(x,y))$ where $f(x,y) = (x+y)^{-2}$, a rational function that is continuous on $\mathbb{R}^2$ except where $x+y=0$, and $g(t) = \tan^{-1} t$, continuous everywhere. Thus G is continuous on its domain $\{(x,y) \mid x+y \neq 0\} = \{(x,y) \mid y \neq -x\}$.

37. $f(x,y) = \begin{cases} \dfrac{x^2y^3}{2x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 1 & \text{if } (x,y) = (0,0) \end{cases}$ The first piece of f is a rational function defined everywhere except at the origin, so f is continuous on $\mathbb{R}^2$ except possibly at the origin. Since $x^2 \leq 2x^2 + y^2$, we have $\left|x^2y^3/(2x^2+y^2)\right| \leq \left|y^3\right|$. We know that $\left|y^3\right| \to 0$ as $(x,y) \to (0,0)$. So, by the Squeeze Theorem, $\lim\limits_{(x,y)\to(0,0)} f(x,y) = \lim\limits_{(x,y)\to(0,0)} \dfrac{x^2y^3}{2x^2+y^2} = 0$. But $f(0,0) = 1$, so f is discontinuous at $(0,0)$. Therefore, f is continuous on the set $\{(x,y) \mid (x,y) \neq (0,0)\}$.