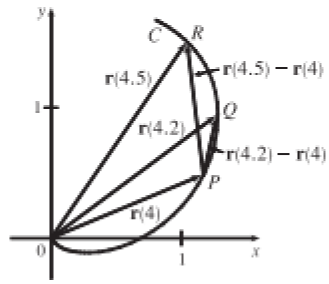
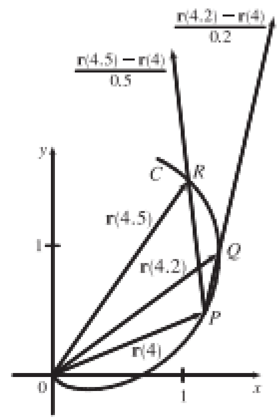
Section 13.2 - 1, 5, 13, 17, 22, 25, 37, 39, 49

1. (a)



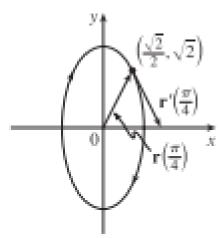
(b) $\dfrac{\mathbf{r}(4.5)-\mathbf{r}(4)}{0.5} = 2[\mathbf{r}(4.5)-\mathbf{r}(4)]$, so we draw a vector in the same direction but with twice the length of the vector $\mathbf{r}(4.5)-\mathbf{r}(4)$.

$\dfrac{\mathbf{r}(4.2)-\mathbf{r}(4)}{0.2} = 5[\mathbf{r}(4.2)-\mathbf{r}(4)]$, so we draw a vector in the same direction but with 5 times the length of the vector $\mathbf{r}(4.2)-\mathbf{r}(4)$.



5. $x = \sin t$, $y = 2\cos t$ so $x^2 + (y/2)^2 = 1$ and the curve is an ellipse.

(a), (c)



(b) $\mathbf{r}'(t) = \cos t\,\mathbf{i} - 2\sin t\,\mathbf{j}$,

$$\mathbf{r}'\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}\,\mathbf{i} - \sqrt{2}\,\mathbf{j}$$

13. $\mathbf{r}(t) = e^{t^2}\,\mathbf{i} - \mathbf{j} + \ln(1+3t)\,\mathbf{k} \quad\Rightarrow\quad \mathbf{r}'(t) = 2te^{t^2}\,\mathbf{i} + \dfrac{3}{1+3t}\,\mathbf{k}$

17. $\mathbf{r}'(t) = \langle -te^{-t} + e^{-t}, 2/(1+t^2), 2e^t\rangle \quad\Rightarrow\quad \mathbf{r}'(0) = \langle 1,2,2\rangle$. So $|\mathbf{r}'(0)| = \sqrt{1^2+2^2+2^2} = \sqrt{9} = 3$ and

$\mathbf{T}(0) = \dfrac{\mathbf{r}'(0)}{|\mathbf{r}'(0)|} = \frac{1}{3}\langle 1,2,2\rangle = \langle \frac{1}{3}, \frac{2}{3}, \frac{2}{3}\rangle$.

22. $\mathbf{r}(t) = \langle e^{2t}, e^{-2t}, te^{2t}\rangle \quad\Rightarrow\quad \mathbf{r}'(t) = \langle 2e^{2t}, -2e^{-2t}, (2t+1)e^{2t}\rangle \quad\Rightarrow\quad \mathbf{r}'(0) = \langle 2e^0, -2e^0, (0+1)e^0\rangle = \langle 2,-2,1\rangle$

and $|\mathbf{r}'(0)| = \sqrt{2^2+(-2)^2+1^2} = 3$. Then $\mathbf{T}(0) = \dfrac{\mathbf{r}'(0)}{|\mathbf{r}'(0)|} = \frac{1}{3}\langle 2,-2,1\rangle = \langle \frac{2}{3}, -\frac{2}{3}, \frac{1}{3}\rangle$.

$\mathbf{r}''(t) = \langle 4e^{2t}, 4e^{-2t}, (4t+4)e^{2t}\rangle \quad\Rightarrow\quad \mathbf{r}''(0) = \langle 4e^0, 4e^0, (0+4)e^0\rangle = \langle 4,4,4\rangle$.

$$\begin{aligned}\mathbf{r}'(t)\cdot\mathbf{r}''(t) &= \langle 2e^{2t}, -2e^{-2t}, (2t+1)e^{2t}\rangle \cdot \langle 4e^{2t}, 4e^{-2t}, (4t+4)e^{2t}\rangle \\ &= (2e^{2t})(4e^{2t}) + (-2e^{-2t})(4e^{-2t}) + ((2t+1)e^{2t})((4t+4)e^{2t}) \\ &= 8e^{4t} - 8e^{-4t} + (8t^2+12t+4)e^{4t} = (8t^2+12t+12)e^{4t} - 8e^{-4t}\end{aligned}$$

25. The vector equation for the curve is $\mathbf{r}(t) = \langle e^{-t}\cos t, e^{-t}\sin t, e^{-t}\rangle$, so

$$\begin{aligned}\mathbf{r}'(t) &= \langle e^{-t}(-\sin t) + (\cos t)(-e^{-t}), e^{-t}\cos t + (\sin t)(-e^{-t}), (-e^{-t})\rangle \\ &= \langle -e^{-t}(\cos t + \sin t), e^{-t}(\cos t - \sin t), -e^{-t}\rangle\end{aligned}$$

The point $(1, 0, 1)$ corresponds to $t = 0$, so the tangent vector there is

$\mathbf{r}'(0) = \langle -e^0(\cos 0 + \sin 0), e^0(\cos 0 - \sin 0), -e^0\rangle = \langle -1, 1, -1\rangle$. Thus, the tangent line is parallel to the vector $\langle -1, 1, -1\rangle$ and parametric equations are $x = 1 + (-1)t = 1 - t$, $y = 0 + 1\cdot t = t$, $z = 1 + (-1)t = 1 - t$.

37. $\int_0^{\pi/2}(3\sin^2 t\,\cos t\,\mathbf{i} + 3\sin t\,\cos^2 t\,\mathbf{j} + 2\sin t\,\cos t\,\mathbf{k})\,dt$

$$\begin{aligned}&= \left(\int_0^{\pi/2} 3\sin^2 t\,\cos t\,dt\right)\mathbf{i} + \left(\int_0^{\pi/2} 3\sin t\,\cos^2 t\,dt\right)\mathbf{j} + \left(\int_0^{\pi/2} 2\sin t\,\cos t\,dt\right)\mathbf{k} \\ &= \left[\sin^3 t\right]_0^{\pi/2}\mathbf{i} + \left[-\cos^3 t\right]_0^{\pi/2}\mathbf{j} + \left[\sin^2 t\right]_0^{\pi/2}\mathbf{k} = (1-0)\,\mathbf{i} + (0+1)\,\mathbf{j} + (1-0)\,\mathbf{k} = \mathbf{i} + \mathbf{j} + \mathbf{k}\end{aligned}$$

39. $\int(\sec^2 t\,\mathbf{i} + t(t^2+1)^3\,\mathbf{j} + t^2\ln t\,\mathbf{k})\,dt = \left(\int \sec^2 t\,dt\right)\mathbf{i} + \left(\int t(t^2+1)^3\,dt\right)\mathbf{j} + \left(\int t^2\ln t\,dt\right)\mathbf{k}$

$$= \tan t\,\mathbf{i} + \tfrac{1}{8}(t^2+1)^4\,\mathbf{j} + \left(\tfrac{1}{3}t^3\ln t - \tfrac{1}{9}t^3\right)\mathbf{k} + \mathbf{C},$$

where $\mathbf{C}$ is a vector constant of integration. [For the z-component, integrate by parts with $u = \ln t$, $dv = t^2\,dt$.]

49. By Formula 4 of Theorem 3, $f'(t) = \mathbf{u}'(t)\cdot\mathbf{v}(t) + \mathbf{u}(t)\cdot\mathbf{v}'(t)$, and $\mathbf{v}'(t) = \langle 1, 2t, 3t^2\rangle$, so

$f'(2) = \mathbf{u}'(2)\cdot\mathbf{v}(2) + \mathbf{u}(2)\cdot\mathbf{v}'(2) = \langle 3, 0, 4\rangle\cdot\langle 2, 4, 8\rangle + \langle 1, 2, -1\rangle\cdot\langle 1, 4, 12\rangle = 6 + 0 + 32 + 1 + 8 - 12 = 35.$