Section 13.1 - 1, 6, 7, 13, 17, 21-26, 47

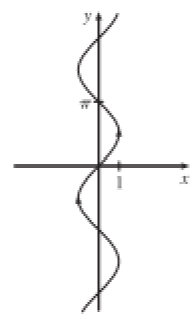
1. The component functions $\sqrt{4-t^2}$, e^{-3t}, and $\ln(t+1)$ are all defined when $4-t^2 \geq 0 \Rightarrow -2 \leq t \leq 2$ and $t+1>0 \Rightarrow t>-1$, so the domain of $\mathbf{r}$ is $(-1, 2]$.

6. $\lim_{t\to\infty} te^{-t} = \lim_{t\to\infty} \frac{t}{e^t} = \lim_{t\to\infty} \frac{1}{e^t} = 0$ [by l'Hospital's Rule], $\lim_{t\to\infty} \frac{t^3+t}{2t^3-1} = \lim_{t\to\infty} \frac{1+(1/t^2)}{2-(1/t^3)} = \frac{1+0}{2-0} = \frac{1}{2}$,

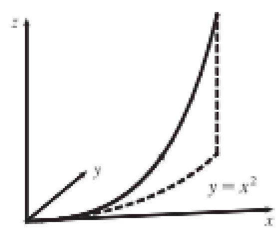
and $\lim_{t\to\infty} t\sin\frac{1}{t} = \lim_{t\to\infty} \frac{\sin(1/t)}{1/t} = \lim_{t\to\infty} \frac{\cos(1/t)(-1/t^2)}{-1/t^2} = \lim_{t\to\infty} \cos\frac{1}{t} = \cos 0 = 1$ [again by l'Hospital's Rule].

Thus $\lim_{t\to\infty} \left\langle te^{-t}, \frac{t^3+t}{2t^3-1}, t\sin\frac{1}{t} \right\rangle = \left\langle 0, \frac{1}{2}, 1 \right\rangle$.

7. The corresponding parametric equations for this curve are $x = \sin t$, $y = t$. We can make a table of values, or we can eliminate the parameter: $t = y \Rightarrow x = \sin y$, with $y \in \mathbb{R}$. By comparing different values of t, we find the direction in which t increases as indicated in the graph.



13. The parametric equations are $x = t^2$, $y = t^4$, $z = t^6$. These are positive for $t \neq 0$ and 0 when $t = 0$. So the curve lies entirely in the first octant. The projection of the graph onto the xy-plane is $y = x^2$, $y > 0$, a half parabola. Onto the xz-plane $z = x^3$, $z > 0$, a half cubic, and the yz-plane, $y^3 = z^2$.



17. Taking $\mathbf{r}_0 = \langle 2, 0, 0\rangle$ and $\mathbf{r}_1 = \langle 6, 2, -2\rangle$, we have from Equation 12.5.4
$\mathbf{r}(t) = (1-t)\,\mathbf{r}_0 + t\,\mathbf{r}_1 = (1-t)\langle 2,0,0\rangle + t\langle 6,2,-2\rangle$, $0 \leq t \leq 1$ or $\mathbf{r}(t) = \langle 2+4t, 2t, -2t\rangle$, $0 \leq t \leq 1$.
Parametric equations are $x = 2+4t$, $y = 2t$, $z = -2t$, $0 \leq t \leq 1$.

21. $x = t\cos t,\ y = t,\ z = t\sin t,\ t \geq 0$. At any point (x, y, z) on the curve, $x^2 + z^2 = t^2\cos^2 t + t^2\sin^2 t = t^2 = y^2$ so the curve lies on the circular cone $x^2 + z^2 = y^2$ with axis the y-axis. Also notice that $y \geq 0$; the graph is II.

22. $x = \cos t,\ y = \sin t,\ z = 1/(1 + t^2)$. At any point on the curve we have $x^2 + y^2 = \cos^2 t + \sin^2 t = 1$, so the curve lies on a circular cylinder $x^2 + y^2 = 1$ with axis the z-axis. Notice that $0 < z \leq 1$ and $z = 1$ only for $t = 0$. A point (x, y, z) on

the curve lies directly above the point $(x, y, 0)$, which moves counterclockwise around the unit circle in the xy-plane as t increases, and $z \to 0$ as $t \to \pm\infty$. The graph must be VI.

23. $x = t,\ y = 1/(1 + t^2),\ z = t^2$. At any point on the curve we have $z = x^2$, so the curve lies on a parabolic cylinder parallel to the y-axis. Notice that $0 < y \leq 1$ and $z \geq 0$. Also the curve passes through $(0, 1, 0)$ when $t = 0$ and $y \to 0$, $z \to \infty$ as $t \to \pm\infty$, so the graph must be V.

24. $x = \cos t,\ y = \sin t,\ z = \cos 2t$. $x^2 + y^2 = \cos^2 t + \sin^2 t = 1$, so the curve lies on a circular cylinder with axis the z-axis. A point (x, y, z) on the curve lies directly above or below $(x, y, 0)$, which moves around the unit circle in the xy-plane with period 2π. At the same time, the z-value of the point (x, y, z) oscillates with a period of π. So the curve repeats itself and the graph is I.

25. $x = \cos 8t,\ y = \sin 8t,\ z = e^{0.8t},\ t \geq 0$. $x^2 + y^2 = \cos^2 8t + \sin^2 8t = 1$, so the curve lies on a circular cylinder with axis the z-axis. A point (x, y, z) on the curve lies directly above the point $(x, y, 0)$, which moves counterclockwise around the unit circle in the xy-plane as t increases. The curve starts at $(1, 0, 1)$, when $t = 0$, and $z \to \infty$ (at an increasing rate) as $t \to \infty$, so the graph is IV.

26. $x = \cos^2 t,\ y = \sin^2 t,\ z = t$. $x + y = \cos^2 t + \sin^2 t = 1$, so the curve lies in the vertical plane $x + y = 1$. x and y are periodic, both with period π, and z increases as t increases, so the graph is III.

47. For the particles to collide, we require $\mathbf{r}_1(t) = \mathbf{r}_2(t) \Leftrightarrow \langle t^2, 7t - 12, t^2 \rangle = \langle 4t - 3, t^2, 5t - 6 \rangle$. Equating components gives $t^2 = 4t - 3$, $7t - 12 = t^2$, and $t^2 = 5t - 6$. From the first equation, $t^2 - 4t + 3 = 0 \Leftrightarrow (t - 3)(t - 1) = 0$ so $t = 1$ or $t = 3$. $t = 1$ does not satisfy the other two equations, but $t = 3$ does. The particles collide when $t = 3$, at the point $(9, 9, 9)$.