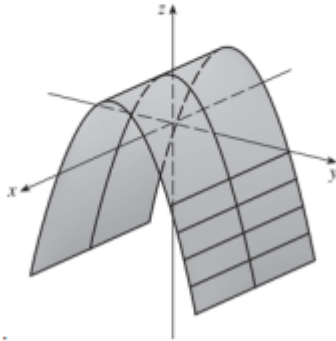
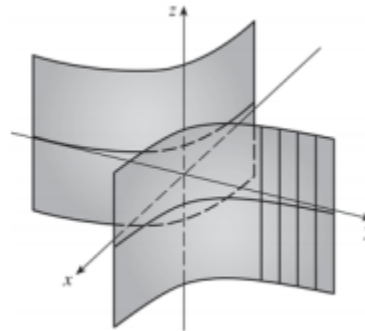


Section 12.6 - 5, 7, 13, 17, 21-28, 33, 35

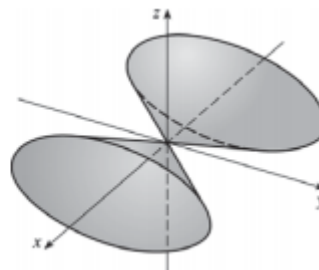
5. Since  $x$  is missing, each vertical trace  $z = 1 - y^2$ ,  $x = k$ , is a copy of the same parabola in the plane  $x = k$ . Thus the surface  $z = 1 - y^2$  is a parabolic cylinder with rulings parallel to the  $x$ -axis.



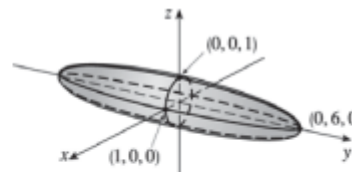
7. Since  $z$  is missing, each horizontal trace  $xy = 1$ ,  $z = k$ , is a copy of the same hyperbola in the plane  $z = k$ . Thus the surface  $xy = 1$  is a hyperbolic cylinder with rulings parallel to the  $z$ -axis.



13.  $x^2 = y^2 + 4z^2$ . The traces in  $x = k$  are the ellipses  $y^2 + 4z^2 = k^2$ . The traces in  $y = k$  are  $x^2 - 4z^2 = k^2$ , hyperbolas for  $k \neq 0$  and two intersecting lines if  $k = 0$ . Similarly, the traces in  $z = k$  are  $x^2 - y^2 = 4k^2$ , hyperbolas for  $k \neq 0$  and two intersecting lines if  $k = 0$ . We recognize the graph as an elliptic cone with axis the  $x$ -axis and vertex the origin.



17.  $36x^2 + y^2 + 36z^2 = 36$ . The traces in  $x = k$  are  $y^2 + 36z^2 = 36(1 - k^2)$ , a family of ellipses for  $|k| < 1$ . (The traces are a single point for  $|k| = 1$  and are empty for  $|k| > 1$ .) The traces in  $y = k$  are the circles  $36x^2 + 36z^2 = 36 - k^2 \iff x^2 + z^2 = 1 - \frac{1}{36}k^2$ ,  $|k| < 6$ , and the traces in  $z = k$  are the ellipses  $36x^2 + y^2 = 36(1 - k^2)$ ,  $|k| < 1$ . The graph is an ellipsoid centered at the origin with intercepts  $x = \pm 1$ ,  $y = \pm 6$ ,  $z = \pm 1$ .



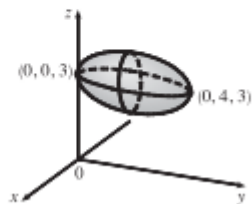
21. This is the equation of an ellipsoid:  $x^2 + 4y^2 + 9z^2 = x^2 + \frac{y^2}{(1/2)^2} + \frac{z^2}{(1/3)^2} = 1$ , with  $x$ -intercepts  $\pm 1$ ,  $y$ -intercepts  $\pm \frac{1}{2}$  and  $z$ -intercepts  $\pm \frac{1}{3}$ . So the major axis is the  $x$ -axis and the only possible graph is VII.
22. This is the equation of an ellipsoid:  $9x^2 + 4y^2 + z^2 = \frac{x^2}{(1/3)^2} + \frac{y^2}{(1/2)^2} + z^2 = 1$ , with  $x$ -intercepts  $\pm \frac{1}{3}$ ,  $y$ -intercepts  $\pm \frac{1}{2}$  and  $z$ -intercepts  $\pm 1$ . So the major axis is the  $z$ -axis and the only possible graph is IV.
23. This is the equation of a hyperboloid of one sheet, with  $a = b = c = 1$ . Since the coefficient of  $y^2$  is negative, the axis of the hyperboloid is the  $y$ -axis, hence the correct graph is II.
24. This is a hyperboloid of two sheets, with  $a = b = c = 1$ . This surface does not intersect the  $xz$ -plane at all, so the axis of the hyperboloid is the  $y$ -axis and the graph is III.
25. There are no real values of  $x$  and  $z$  that satisfy this equation for  $y < 0$ , so this surface does not extend to the left of the  $xz$ -plane. The surface intersects the plane  $y = k > 0$  in an ellipse. Notice that  $y$  occurs to the first power whereas  $x$  and  $z$  occur to the second power. So the surface is an elliptic paraboloid with axis the  $y$ -axis. Its graph is VI.
26. This is the equation of a cone with axis the  $y$ -axis, so the graph is I.
27. This surface is a cylinder because the variable  $y$  is missing from the equation. The intersection of the surface and the  $xz$ -plane is an ellipse. So the graph is VIII.
28. This is the equation of a hyperbolic paraboloid. The trace in the  $xy$ -plane is the parabola  $y = x^2$ . So the correct graph is V.

33. Completing squares in  $y$  and  $z$  gives

$$4x^2 + (y - 2)^2 + 4(z - 3)^2 = 4 \text{ or}$$

$$x^2 + \frac{(y - 2)^2}{4} + (z - 3)^2 = 1, \text{ an ellipsoid with}$$

center  $(0, 2, 3)$ .



35. Completing squares in all three variables gives

$$(x - 2)^2 - (y + 1)^2 + (z - 1)^2 = 0 \text{ or}$$

$$(y + 1)^2 = (x - 2)^2 + (z - 1)^2, \text{ a circular cone with}$$

center  $(2, -1, 1)$  and axis the horizontal line  $x = 2$ ,

$z = 1$ .

