Section 12.5 - 1, 5, 9, 16, 21, 25, 29, 35, 41, 46, 51, 55, 69

1. (a) True; each of the first two lines has a direction vector parallel to the direction vector of the third line, so these vectors are each scalar multiples of the third direction vector. Then the first two direction vectors are also scalar multiples of each other, so these vectors, and hence the two lines, are parallel.

   (b) False; for example, the x- and y-axes are both perpendicular to the z-axis, yet the x- and y-axes are not parallel.

   (c) True; each of the first two planes has a normal vector parallel to the normal vector of the third plane, so these two normal vectors are parallel to each other and the planes are parallel.

   (d) False; for example, the xy- and yz-planes are not parallel, yet they are both perpendicular to the xz-plane.

   (e) False; the x- and y-axes are not parallel, yet they are both parallel to the plane $z = 1$.

   (f) True; if each line is perpendicular to a plane, then the lines' direction vectors are both parallel to a normal vector for the plane. Thus, the direction vectors are parallel to each other and the lines are parallel.

   (g) False; the planes $y = 1$ and $z = 1$ are not parallel, yet they are both parallel to the x-axis.

   (h) True; if each plane is perpendicular to a line, then any normal vector for each plane is parallel to a direction vector for the line. Thus, the normal vectors are parallel to each other and the planes are parallel.

   (i) True; see Figure 9 and the accompanying discussion.

   (j) False; they can be skew, as in Example 3.

   (k) True. Consider any normal vector for the plane and any direction vector for the line. If the normal vector is perpendicular to the direction vector, the line and plane are parallel. Otherwise, the vectors meet at an angle θ, $0^\circ \leq \theta < 90^\circ$, and the line will intersect the plane at an angle $90^\circ - \theta$.

5. A line perpendicular to the given plane has the same direction as a normal vector to the plane, such as $\mathbf{n} = \langle 1, 3, 1 \rangle$. So $\mathbf{r}_0 = \mathbf{i} + 6\,\mathbf{k}$, and we can take $\mathbf{v} = \mathbf{i} + 3\,\mathbf{j} + \mathbf{k}$. Then a vector equation is $\mathbf{r} = (\mathbf{i} + 6\,\mathbf{k}) + t(\mathbf{i} + 3\,\mathbf{j} + \mathbf{k}) = (1+t)\,\mathbf{i} + 3t\,\mathbf{j} + (6+t)\,\mathbf{k}$, and parametric equations are $x = 1 + t$, $y = 3t$, $z = 6 + t$.

9. $\mathbf{v} = \langle 3 - (-8), -2 - 1, 4 - 4 \rangle = \langle 11, -3, 0 \rangle$, and letting $P_0 = (-8, 1, 4)$, parametric equations are $x = -8 + 11t$, $y = 1 - 3t$, $z = 4 + 0t = 4$, while symmetric equations are $\dfrac{x+8}{11} = \dfrac{y-1}{-3}$, $z = 4$. Notice here that the direction number $c = 0$, so rather than writing $\dfrac{z-4}{0}$ in the symmetric equation we must write the equation $z = 4$ separately.

16. (a) A vector normal to the plane $x - y + 3z = 7$ is $\mathbf{n} = \langle 1, -1, 3 \rangle$, and since the line is to be perpendicular to the plane, $\mathbf{n}$ is also a direction vector for the line. Thus parametric equations of the line are $x = 2 + t$, $y = 4 - t$, $z = 6 + 3t$.

    (b) On the xy-plane, $z = 0$. So $z = 6 + 3t = 0 \;\Rightarrow\; t = -2$ in the parametric equations of the line, and therefore $x = 0$ and $y = 6$, giving the point of intersection $(0, 6, 0)$. For the yz-plane, $x = 0$ so we get the same point of interesection: $(0, 6, 0)$. For the xz-plane, $y = 0$ which implies $t = 4$, so $x = 6$ and $z = 18$ and the point of intersection is $(6, 0, 18)$.

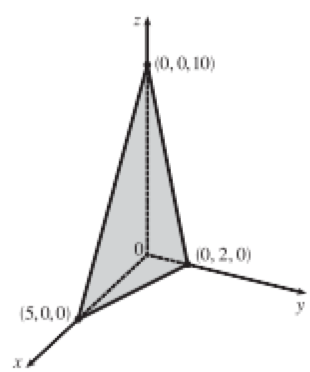
21. Since the direction vectors $\langle 1, -2, -3\rangle$ and $\langle 1, 3, -7\rangle$ aren't scalar multiples of each other, the lines aren't parallel. Parametric equations of the lines are L_1: $x = 2 + t$, $y = 3 - 2t$, $z = 1 - 3t$ and L_2: $x = 3 + s$, $y = -4 + 3s$, $z = 2 - 7s$. Thus, for the lines to intersect, the three equations $2 + t = 3 + s$, $3 - 2t = -4 + 3s$, and $1 - 3t = 2 - 7s$ must be satisfied simultaneously. Solving the first two equations gives $t = 2$, $s = 1$ and checking, we see that these values do satisfy the third equation, so the lines intersect when $t = 2$ and $s = 1$, that is, at the point $(4, -1, -5)$.

25. $\mathbf{i} + 4\mathbf{j} + \mathbf{k} = \langle 1, 4, 1\rangle$ is a normal vector to the plane and $\left(-1, \frac{1}{2}, 3\right)$ is a point on the plane, so setting $a = 1$, $b = 4$, $c = 1$, $x_0 = -1$, $y_0 = \frac{1}{2}$, $z_0 = 3$ in Equation 7 gives $1[x - (-1)] + 4\left(y - \frac{1}{2}\right) + 1(z - 3) = 0$ or $x + 4y + z = 4$ as an equation of the plane.

29. Since the two planes are parallel, they will have the same normal vectors. So we can take $\mathbf{n} = \langle 1, 1, 1\rangle$, and an equation of the plane is $1(x - 1) + 1\left(y - \frac{1}{2}\right) + 1\left(z - \frac{1}{3}\right) = 0$ or $x + y + z = \frac{11}{6}$ or $6x + 6y + 6z = 11$.

35. If we first find two nonparallel vectors in the plane, their cross product will be a normal vector to the plane. Since the given line lies in the plane, its direction vector $\mathbf{a} = \langle -2, 5, 4\rangle$ is one vector in the plane. We can verify that the given point $(6, 0, -2)$ does not lie on this line, so to find another nonparallel vector $\mathbf{b}$ which lies in the plane, we can pick any point on the line and find a vector connecting the points. If we put $t = 0$, we see that $(4, 3, 7)$ is on the line, so $\mathbf{b} = \langle 6 - 4, 0 - 3, -2 - 7\rangle = \langle 2, -3, -9\rangle$ and $\mathbf{n} = \mathbf{a} \times \mathbf{b} = \langle -45 + 12, 8 - 18, 6 - 10\rangle = \langle -33, -10, -4\rangle$. Thus, an equation of the plane is $-33(x - 6) - 10(y - 0) - 4[z - (-2)] = 0$ or $33x + 10y + 4z = 190$.

41. To find the x-intercept we set $y = z = 0$ in the equation $2x + 5y + z = 10$ and obtain $2x = 10 \;\Rightarrow\; x = 5$ so the x-intercept is $(5, 0, 0)$. When $x = z = 0$ we get $5y = 10 \;\Rightarrow\; y = 2$, so the y-intercept is $(0, 2, 0)$. Setting $x = y = 0$ gives $z = 10$, so the z-intercept is $(0, 0, 10)$ and we graph the portion of the plane that lies in the first octant.



46. Substitute the parametric equations of the line into the equation of the plane: $(1 + 2t) + 2(4t) - (2 - 3t) + 1 = 0 \;\Rightarrow\; 13t = 0 \;\Rightarrow\; t = 0$. Therefore, the point of intersection of the line and the plane is given by $x = 1 + 2(0) = 1$, $y = 4(0) = 0$, and $z = 2 - 3(0) = 2$, that is, the point $(1, 0, 2)$.

51. Normal vectors for the planes are $\mathbf{n}_1 = \langle 1, 4, -3\rangle$ and $\mathbf{n}_2 = \langle -3, 6, 7\rangle$, so the normals (and thus the planes) aren't parallel. But $\mathbf{n}_1 \cdot \mathbf{n}_2 = -3 + 24 - 21 = 0$, so the normals (and thus the planes) are perpendicular.

55. The normals are $\mathbf{n}_1 = \langle 1, -4, 2 \rangle$ and $\mathbf{n}_2 = \langle 2, -8, 4 \rangle$. Since $\mathbf{n}_2 = 2\mathbf{n}_1$, the normals (and thus the planes) are parallel.

69. Let $Q = (1, 3, 4)$ and $R = (2, 1, 1)$, points on the line corresponding to $t = 0$ and $t = 1$. Let

$P = (4, 1, -2)$. Then $\mathbf{a} = \overrightarrow{QR} = \langle 1, -2, -3 \rangle$, $\mathbf{b} = \overrightarrow{QP} = \langle 3, -2, -6 \rangle$. The distance is

$$d = \frac{|\mathbf{a} \times \mathbf{b}|}{|\mathbf{a}|} = \frac{|\langle 1, -2, -3 \rangle \times \langle 3, -2, -6 \rangle|}{|\langle 1, -2, -3 \rangle|} = \frac{|\langle 6, -3, 4 \rangle|}{|\langle 1, -2, -3 \rangle|} = \frac{\sqrt{6^2 + (-3)^2 + 4^2}}{\sqrt{1^2 + (-2)^2 + (-3)^2}} = \frac{\sqrt{61}}{\sqrt{14}} = \sqrt{\frac{61}{14}}.$$