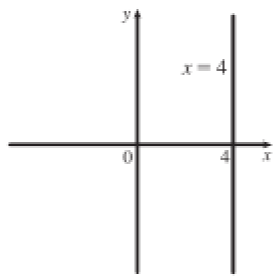
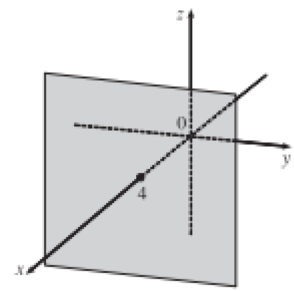
Section 12.1 - 3, 6, 7, 13, 17, 21, 27-33, 35, 37

3. The distance from a point to the yz-plane is the absolute value of the x-coordinate of the point. $C(2, 4, 6)$ has the x-coordinate with the smallest absolute value, so C is the point closest to the yz-plane. $A(-4, 0, -1)$ must lie in the xz-plane since the distance from A to the xz-plane, given by the y-coordinate of A, is 0.

6. (a) In $\mathbb{R}^2$, the equation $x = 4$ represents a line parallel to the y-axis. In $\mathbb{R}^3$, the equation $x = 4$ represents the set $\{(x, y, z) \mid x = 4\}$, the set of all points whose x-coordinate is 4. This is the vertical plane that is parallel to the yz-plane and 4 units in front of it.





7. We can find the lengths of the sides of the triangle by using the distance formula between pairs of vertices:

$$|PQ| = \sqrt{(7-3)^2 + [0-(-2)]^2 + [1-(-3)]^2} = \sqrt{16+4+16} = 6$$
$$|QR| = \sqrt{(1-7)^2 + (2-0)^2 + (1-1)^2} = \sqrt{36+4+0} = \sqrt{40} = 2\sqrt{10}$$
$$|RP| = \sqrt{(3-1)^2 + (-2-2)^2 + (-3-1)^2} = \sqrt{4+16+16} = 6$$

The longest side is QR, but the Pythagorean Theorem is not satisfied: $|PQ|^2 + |RP|^2 \neq |QR|^2$. Thus PQR is not a right triangle. PQR is isosceles, as two sides have the same length.

13. The radius of the sphere is the distance between $(4, 3, -1)$ and $(3, 8, 1)$: $r = \sqrt{(3-4)^2 + (8-3)^2 + [1-(-1)]^2} = \sqrt{30}$. Thus, an equation of the sphere is $(x-3)^2 + (y-8)^2 + (z-1)^2 = 30$.

17. Completing squares in the equation $2x^2 - 8x + 2y^2 + 2z^2 + 24z = 1$ gives $2(x^2 - 4x + 4) + 2y^2 + 2(z^2 + 12z + 36) = 1 + 8 + 72 \quad \Rightarrow \quad 2(x-2)^2 + 2y^2 + 2(z+6)^2 = 81 \quad \Rightarrow$ $(x-2)^2 + y^2 + (z+6)^2 = \frac{81}{2}$, which we recognize as an equation of a sphere with center $(2, 0, -6)$ and radius $\sqrt{\frac{81}{2}} = 9/\sqrt{2}$.

21. (a) Since the sphere touches the xy-plane, its radius is the distance from its center, $(2, -3, 6)$, to the xy-plane, namely 6. Therefore $r = 6$ and an equation of the sphere is $(x-2)^2 + (y+3)^2 + (z-6)^2 = 6^2 = 36$.

(b) The radius of this sphere is the distance from its center $(2, -3, 6)$ to the yz-plane, which is 2. Therefore, an equation is $(x-2)^2 + (y+3)^2 + (z-6)^2 = 4$.

(c) Here the radius is the distance from the center $(2, -3, 6)$ to the xz-plane, which is 3. Therefore, an equation is $(x-2)^2 + (y+3)^2 + (z-6)^2 = 9$.

27. The inequality $0 \leq z \leq 6$ represents all points on or between the horizontal planes $z = 0$ (the xy-plane) and $z = 6$.

28. The equation $z^2 = 1 \quad \Leftrightarrow \quad z = \pm 1$ represents two horizontal planes; $z = 1$ is parallel to the xy-plane, one unit above it, and $z = -1$ is one unit below it.

29. Because $z = -1$, all points in the region must lie in the horizontal plane $z = -1$. In addition, $x^2 + y^2 = 4$, so the region consists of all points that lie on a circle with radius 2 and center on the z-axis that is contained in the plane $z = -1$.

30. Here $y^2 + z^2 = 16$ with no restrictions on x, so a point in the region must lie on a circle of radius 4, center on the x-axis, but it could be in any vertical plane $x = k$ (parallel to the yz-plane). Thus the region consists of all possible circles $y^2 + z^2 = 16$, $x = k$ and is therefore a circular cylinder with radius 4 whose axis is the x-axis.

31. The inequality $x^2 + y^2 + z^2 \leq 3$ is equivalent to $\sqrt{x^2 + y^2 + z^2} \leq \sqrt{3}$, so the region consists of those points whose distance from the origin is at most $\sqrt{3}$. This is the set of all points on or inside the sphere with radius $\sqrt{3}$ and center $(0, 0, 0)$.

32. The equation $x = z$ represents a plane perpendicular to the xz-plane and intersecting the xz-plane in the line $x = z$, $y = 0$.

33. Here $x^2 + z^2 \leq 9$ or equivalently $\sqrt{x^2 + z^2} \leq 3$ which describes the set of all points in $\mathbb{R}^3$ whose distance from the y-axis is at most 3. Thus, the inequality represents the region consisting of all points on or inside a circular cylinder of radius 3 with axis the y-axis.

35. This describes all points whose x-coordinate is between 0 and 5, that is, $0 < x < 5$.

37. This describes a region all of whose points have a distance to the origin which is greater than r, but smaller than R. So inequalities describing the region are $r < \sqrt{x^2 + y^2 + z^2} < R$, or $r^2 < x^2 + y^2 + z^2 < R^2$.