Section 16.9 - The Divergence Theorem

Stoke's Theorem allows us to write a ________ of a ________ as a ________ of a ________.

Want to be able to write a ________ of a ________ as a ________ of a ________.

$d\vec{S} =$ Extension of $ds =$

$\int_C \vec{F} \cdot d\vec{r} =$

Now would like

$\int_C \vec{F} \cdot \vec{n} \, ds =$

The Divergence Theorem:

- E simple solid region
- $S = \partial E$ with positive orientation
- $\vec{F}$ components with continuous partials on open region containing E

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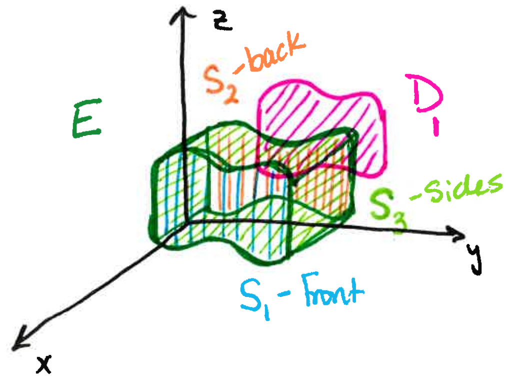


Proof Overview: $\vec{F} = \langle P, Q, R \rangle$

Assume:

$$E = \{(x,y,z) \mid (y,z) \in D_1 \quad g_1(y,z) \le x \le g_2(y,z)\}$$
$$= \{(x,y,z) \mid (x,z) \in D_2 \quad h_1(x,z) \le y \le h_2(x,z)\}$$
$$= \{(x,y,z) \mid (x,y) \in D_3 \quad k_1(x,y) \le z \le k_2(x,y)\}$$

$$\iint_S \vec{F} \cdot d\vec{S} =$$

$$\iiint_E \operatorname{div} \vec{F} \, dV =$$

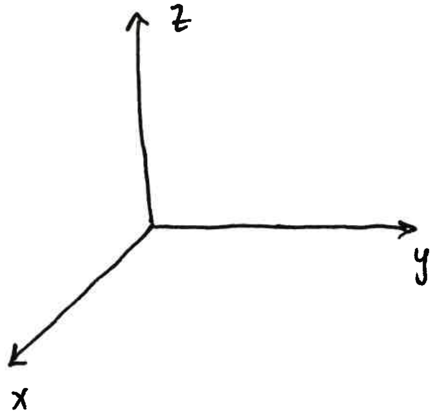
Enough to show:

$$\iiint_E$$

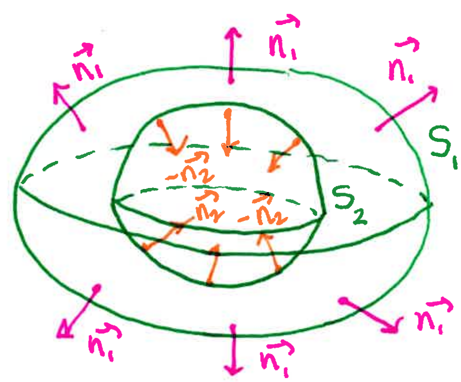
$$\iint_S$$

$\boxed{\text{Example}}$ Find the Flux of the vector field $\vec{F} = \langle x, y, z \rangle$ over the unit sphere.

Example Evaluate $\iint_S \vec{F} \cdot d\vec{S}$ where $\vec{F} = \langle xy, (y^2 + e^{xz^2}), \sin(xy) \rangle$ and S is the surface of E bounded by $z = 1 - x^2$, $z = 0$, $y = 0$ and $y + z = 2$.



- Hallow Solids: $\partial E = S = S_1 \cup S_2$



Normal to E is $\vec{n} =$

$\iiint_E \operatorname{div} \vec{F} \, dV =$