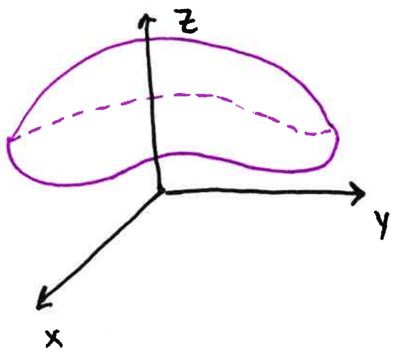
Section 16.8 - Stoke's Theorem

MVC

★ Green's Theorem for vector Functions of 3-variables

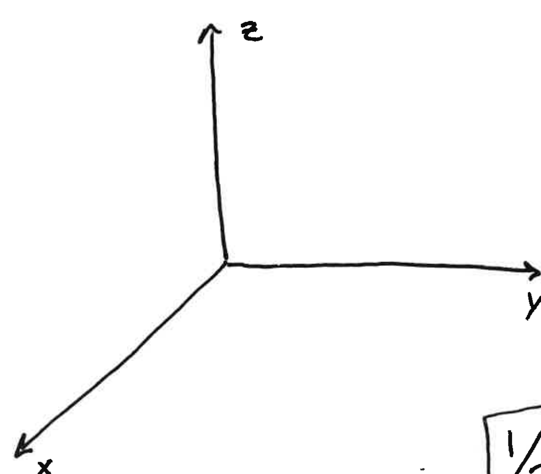
- Green's theorem relates ________ over domain ______
  to ________ over boundary ______
- Stoke's Theorem relates ______ over Surface ______
  to ________ over boundary ______

Important:



Stoke's Theorem

- S Oriented Piecewise-smooth Surface
- $C = \partial S$ Simple, closed, piecewise-smooth, positive orientation
- $\vec{F}$ vector field, components having continuous partials on Open region of $\mathbb{R}^3$ containing S



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Proof (special case): $S: z = g(x,y)$ with $(x,y) \in D \subseteq \mathbb{R}^2$

$C = \partial S \subseteq \mathbb{R}^3$ and $C_1 = \partial D \subseteq \mathbb{R}^2$

$\vec{F} = \langle P, Q, R \rangle$ $\quad$ $\text{curl}\,\vec{F} =$

$\vec{n} =$

$\iint_S \text{curl}\,\vec{F} \cdot d\vec{S} =$

$\int_C \vec{F} \cdot d\vec{r} =$

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Ex. Evaluate $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = \langle -y^2, x, z^2 \rangle$ and C is the curve of intersection of $y+z=2$ and $x^2+y^2=1$; orient C to be counterclockwise when viewed from above.

Ex. Use Stoke's Theorem to compute the integral $\iint_S \text{curl}\,\vec{F} \cdot d\vec{S}$ where $\vec{F} = \langle xz, yz, xy \rangle$ and S: $x^2+y^2+z^2=4$ and $x^2+y^2=1$ above xy-plane.