Section 16.7 - Surface Integrals of Vector Fields

MVC

- Applications:

  ① Gravitational & Pressure Forces

  ② Fluid Flow/mass Flow across a surface

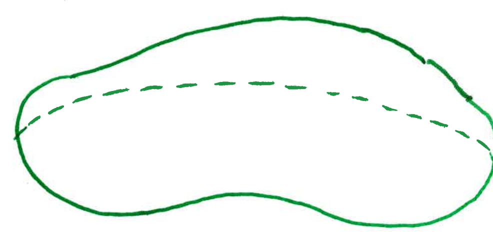
  ③ Electric charge & Electric Fields

Recap:

	Scalar Functions	Vector Functions
Line Integrals		
Surface Integrals		

• Orientation of Surfaces

★ Make a Möbius strip - Color each side a different color

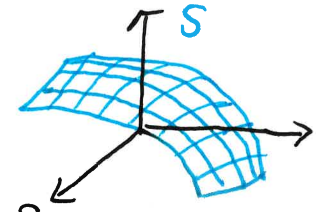


- S is orientable if

- S has an orientation when

	Open Surface	Closed Surface
Positive Orientation:		
Negative Orientation:		

Section 16.7 - Surface Integrals of Vector Functions



- Fluid Flow Motivation:

  Fluid with density ρ and velocity field $\vec{V}$ flowing through S

  Rate of flow per unit area:

  Mass of fluid per unit time crossing S in direction $\vec{n}$:

  Rate of flow through S:

- Surface Integrals of Vector Fields:

  $\vec{F}$ continuous, defined on an oriented surface S with unit normal $\vec{n}$

  then the surface integral of $\vec{F}$ over S:

- S parametrized by $\vec{r}(u,v)$ then:

- S given by $z = g(x,y)$ then:

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Example Find the flux of the vector field $\vec{F} = \langle z, y, x \rangle$ across the Sphere $S: x^2 + y^2 + z^2 = 1$

Example Evaluate $\iint_S \vec{F} \cdot d\vec{S}$ where $\vec{F} = \langle y, x, z \rangle$ and $S: z = 1 - x^2 - y^2$ and $z = 0$